Objective C (Financial Risk Management)

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SUMMARY OF READINGS

- **Coval, Jurek, and Stafford (The Economics of Structured Finance)**

  The CJS paper provides some background information for the causes of the Great Recession of 2008. Structured finance instruments, such as the **collateralized mortgage obligations** mentioned earlier in the BKM readings, played a big role in that market collapse. Those types of instruments are described in some detail here.

- **Cummins (CAT Bond and Other Risk-Linked Securities)**

  The Cummins paper details alternative options to the traditional reinsurance structure, primarily **catastrophe bonds**.

- **Butsic (Solvency Measurement for Property-Liability Risk-Based Capital Applications)**

  Butsic outlines a methodology for establishing risk-based capital, namely expected policyholder deficit.

The next three papers all focus on describing different methods of allocating capital across lines of business. Goldfarb's paper is more all-encompassing than the other two – it also includes background information about what capital is, how it may be established capital at a company-wide level, and the purpose and consequences of allocating capital to different lines of business.

- **Goldfarb (Risk-Adjusted Performance Measurement for P&C Insurers)**
- **Bodoff (Capital Allocation by Percentile Layer)**
- **Cummins (Allocation of Capital in the Insurance Industry)**

Coval, Jurek, and Stafford (The Economics of Structured Finance)

This paper presents a number of causes of the structured finance market collapse in 2008. It is easily one of the more interesting reads in the exam material. From the paper's abstract: "This paper investigates the spectacular rise and fall of structured finance." This introduction is every bit as awesome as the paper itself.

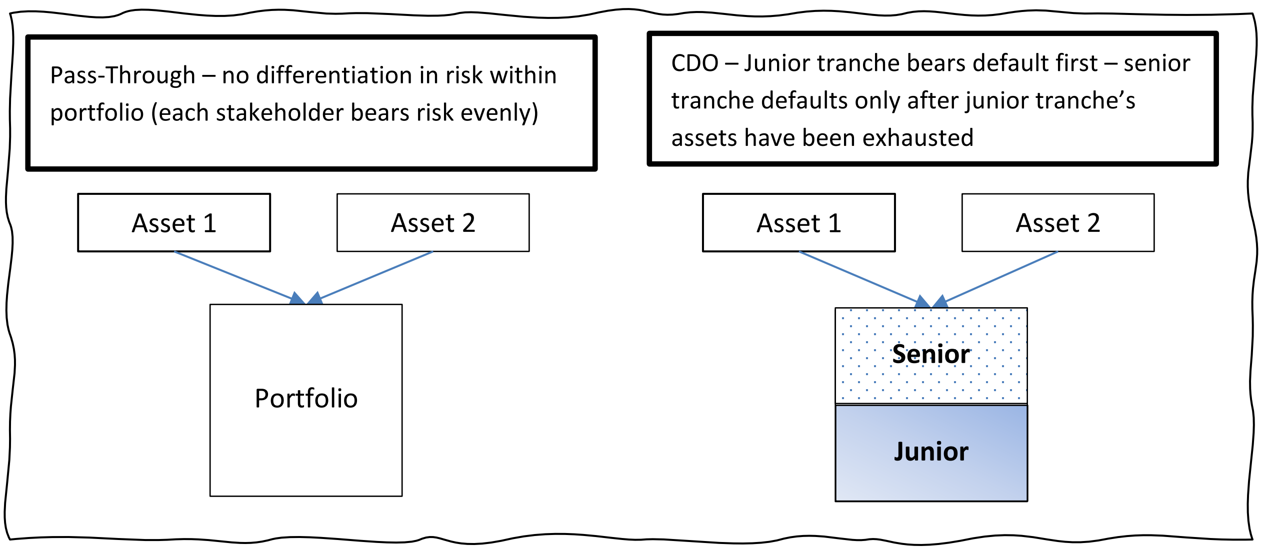
Structured Finance

Structured finance involves two parts: pooling and tranching. In the pooling step, a large number of assets are pooled into a portfolio. This portfolio is called a **special purpose vehicle (SPV)**. The SPV is separate from the lender of the original assets so that if the parent company goes bankrupt, the SPV is still obliged to fulfill payments.

Because of the use of tranching, structured finance is different from a **pass-through securitization**, in which a set of assets (like mortgages) are pooled together and then sold to investors in the open market. A pass-through essentially is structured like a bond. Each investor in the asset-backed security receives a dividend based on the mortgage payments. The expected loss in this portfolio is equal to the average expected loss on the underlying securities, so the credit rating is just given by the average rating.

The second step (tranching) is what differentiates structured finance from pass-through securitization. In structured finance, the investors do not equally share the burden of default. Instead, they are broken into tranches, and the tranches are prioritized in sharing the loss burden. The lowest tranche (junior tranche) suffers loss first, while the highest (senior) tranche only incurs a loss when the junior tranche(s) have been fully exhausted. The junior tranches therefore serve as a protection to the value of the senior tranches, a setup known as **overcollaterization**. It is this differentiation in default risk that would determine how each tranche should be rated. The combination of several risks into a structured asset is known as a **collaterized debt obligation (CDO)**.

The paper presents a straightforward example: It takes two assets, each of value \$1, offering binary payments (1 or 0). We can pool the two assets into a portfolio with notional value \$2, and then separate the portfolio into a junior tranche and senior tranche each paying \$1 of the notional value, with the junior tranche failing to pay (default) if at least \$1 is lost, or when either or both of the assets default. The senior tranche will default only if both assets default (if only one defaults, then the lost dollar will be fully borne by the junior tranche).



The key difference between this and a pass-through is that in a pass-through, the investors are not "junior" and "senior" – if exactly one asset defaults, each investor would lose half of their payoff, and if both defaulted, each investor would lose the entire payoff.

The expected value of each of these tranches is given by the probability of default. Suppose the probability of each of the underlying assets defaulting is 10%. We calculate the tranche probability of default in two opposite circumstances:

		Default probability when…	
Tranche	**When it defaults**	**Assets are totally independent**	**Assets are perfectly correlated**
Junior	When 1 or 2 of the underlying assets defaults	$1 - P(0 \text{ defaults})$ $= 1 - (90\%)^2 = 19\%$	Either both default or neither default, so 10%
Senior	When both of the underlying assets default	$(10\%)^2 = 1\%$	

The key idea is that **the value of each tranche is highly sensitive to the correlation assumption**. When the underlying assets are totally independent, the junior tranche is 19 times more likely to default than is the senior tranche. On the other end of the spectrum, if the assets are totally correlated, each tranche is equally valuable, and this is simply a pass-through! In one case, the junior tranche should offer a much higher return than the senior tranche.

Of course in the structured finance market, we don't just use one or two underlying securities. The paper extends the example to three securities to show how the differences can be magnified even with few risks. So, suppose we have three assets instead. This is summarized in the table below:

		Default probability when…	
Tranche	**When it defaults**	**Assets are totally independent**	**Assets are perfectly correlated**
Junior	1, 2, or 3 assets default	$1-(90\%)^3 = 27.1\%$	10%
Mezzanine	2 or 3 assets default	$P(2)+P(3) = \binom{3}{2}(0.9)(0.1)^2 + (0.1)^3 = 2.8\%$	
Senior	All 3 assets default	$P(3) = (0.1)^3 = 0.1\%$	

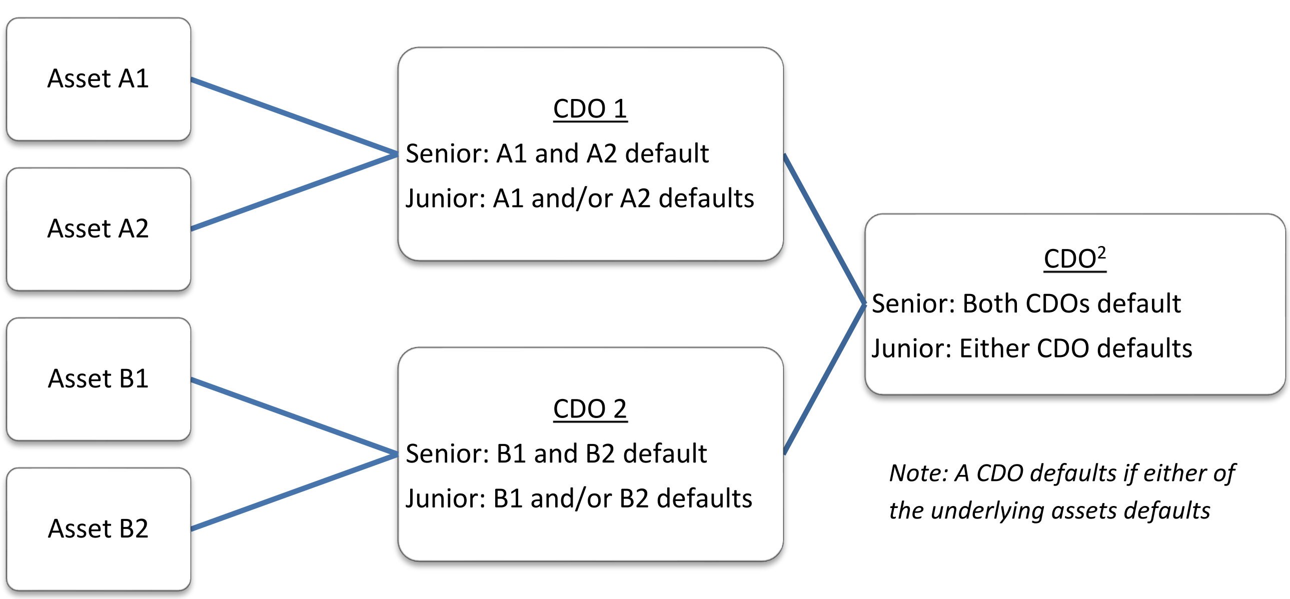
In this case (assuming independence), the junior tranche is significantly riskier, while the other two tranches are less risky than each of the underlying assets.

Now, we could (and shall) further complicate things by adding another round of securitization, as diagramed below. Here, the senior tranche of the **CDO^2 (or synthetic CDO)** defaults when there is a default in both of the underlying CDOs, which occurs when at least one of the assets composing each CDO defaults.

In the CDO^2 (assuming independence), the junior tranche defaults when at least one asset defaults (since that would cause a CDO to default). The probability of a loss in the junior tranche of the CDO^2 is $1-(90\%)^4 = 34.39\%$.

For the senior tranche, we need either Asset A1 or Asset A2 to default (to cause a default in CDO 1), and for at least one of the "B" assets to default. We have a loss in the senior tranche when both of the underlying CDOs default: $(1-(0.9)^2)^2 = 3.61\%$

Underlying Securities	Default Situation	Default Situation



Hopefully the examples have shown that the value of the tranche is critically dependent upon the assumption regarding correlation, because it's a pretty big foreshadow of the point of the paper.

Rating Structured Finance Assets

Credit ratings are intended to be a measure of the likelihood of an issuer to meet payment obligations. A bond is considered **investment grade** if it is rated as BBB- or higher, while **speculative grade** bonds of BB+ rating or below are considered highly likely to default. The text notes that in the model Fitch uses for rating, investment grade bonds fall into ten categories, but with a very small range of default (0.02 to 0.75 percent in annualized default probability). On the other side, the 10 categories of speculative bond ratings encompass a range from a 1.07 to 29.96 percent probability of default.

Sensitivity to Correlation Assumptions

When rating a single asset, the interdependence and correlation of securities is not considered. However, we have seen that CDOs are structured such that the degree of correlation between securities is quite relevant. Even a very small level of imprecision in the estimate creates a large change in the value of a tranche, particularly within the higher tranches. The text simulates payoffs for a large number of underlying assets and summarizes the results in Tables 2 and 3 in the text, reproduced below.

Summary Statistics for CDO and CDO^2 Tranches[1]

	Attachment Points	Default Probability	Expected Payoff	Rating (baseline)	Rating (60% correlation)	Rating (10% prob. Default)
CDO						
Junior	0 – 6%	97.52%	0.59	NR	C	NR
Mezzanine	6 – 12%	2.07%	>0.99	BBB-	B+	CCC
Senior	12 – 100%	<0.00%	>0.99	AAA	BBB-	A+
CDO^2						
Junior	0 – 6%	56.94%	0.93	C	NR	NR
Mezzanine	6 – 12%	<0.00%	>0.99	AAA	C	NR
Senior	12 – 100%	<0.00%	>0.99	AAA	AAA	AAA

In the baseline simulation, the authors found that, using a default correlation of 20% and an individual default probability of 5%, the junior tranche of the CDO was quite a toxic asset, as was that of the CDO^2, while the higher level tranches had lower default probabilities than any of the underlying securities. When the default correlation was made to vary, the authors illustrated the same point identified earlier – **an increase in correlation has the effect of shifting risk up the tranches.**

[1] Joshua Coval, J. J. (2009). The Economics of Structured Finance. *Journal of Economic Perspectives*, pages 11 and 15.

Sensitivity to Default Assumptions

The authors also simulated the sensitivity of each tranche's projected value to the assumed 5% probability of default. They found that within the CDO, the junior tranche bears the biggest loss if the default probability is understated (an increase in default likelihood from 5 to 10 percent created a 55 percent decline in the junior tranche payoff). On the other hand, the senior tranche only realized a 0.01 percent decline in payoff under the circumstances. The same event is observed within the CDO^2 asset, although there is a much more amplified decrease in value for the tranches.

Structured Finance and Subprime Mortgages

Government-sponsored agencies like Fannie Mae, Freddie Mac, and Ginnie Mac are tasked with the role of encouraging home ownership by guaranteeing mortgages written to borrowers meeting certain credit requirements. This allows banks to write mortgages without much exposure to default risk. Once these agencies purchase the mortgages, the mortgages are pooled into mortgage-backed securities and then resold, essentially as a risk-free asset.

Mortgages that do not meet the requirements for government backing may also be packaged and sold by banks directly to investors. These "subprime" mortgage packages exploded in popularity between 1996 and 2006, simultaneous with the general decline in average credit quality of the underlying mortgages. The subprime mortgage market suffered from the double whammy of typically high rates of default (low credit quality) as well as high levels of correlation (concentrated into certain geographic areas). The result played out like a (financial) horror movie:

1. Concentrations of risks created higher-than-projected default correlation levels.
2. Default and recovery aspects are more pessimistic than projected due to decline in credit quality of subprime market and a push to sell a large number of defaulted properties, driving down prices.
3. The popularity of CDO^2 created a large market in which the above effects would be even more pronounced.

The Effect of Systematic Risk in Structured Products

Turning back to the CAPM, we expect securities that are more correlated to the market as a whole to offer higher expected returns than those securities offering the same payoffs but with low correlation to the market. Credit ratings only provide insight as to likelihood of a security's payoff, without consideration of systematic exposures. In theory then, securities with a given credit rating can have a wide variety of yield spreads. For example, the payoff on a catastrophe bond is independent of the economic atmosphere (assuming a catastrophe does not have a material impact on the world economy). On the other end of the spectrum, the security with maximum exposure to systematic risk is the **digital call option** on the stock market.

The digital call option is a binary option that either pays 1 or 0 depending on whether or not the market is at a certain level (the strike price). This option essentially captures the systematic risk of the market, and investors will demand a high return for fully bearing systematic risk. The authors liken structured finance objects to a digital call option – the pooling process creates a diversified set of risks and as the number of assets grows, the remaining exposure in the senior levels is just systematic.

The holders of senior tranches of CDOs are essentially identical to the investors in a digital call option on the market – they are financing systematic risk, as their loss likelihood increases when default correlations increase (overall market tanks). This is riskier than holding a single asset, where default is more related to the firm itself doing poorly. An investor in a senior tranche of a CDO may not appreciate the difference in risk between that and a similarly-rated individual security, and would be attracted to the CDO since it offers a higher-yield for the same degree of "risk" (if using rating as a proxy for risk).

The Rise and Fall of the Structured Finance Market

So, what caused such a spectacular change in the market? Basically, it comes down to how structured finance objects tend to magnify errors in default and correlation assumptions. The authors highlight several factors:

1. CDOs with high ratings (senior tranches) offered higher yields than similarly-rated objects, although not as high as they should have, considering that:
    a. Ratings did not consider the possibility of an economic downturn.
    b. Yields did not consider sensitivity to systematic risk.
2. Because the senior tranches were essentially overvalued, the junior tranches correspondingly were undervalued, which made them appear even more attractive.
3. CDO^2 structures boomed in popularity to meet demand for all tranches.

This clash of issues could have been caused by a combination of factors. For one, credit agencies could genuinely have been unaware of the impact of certain assumptions on default risk, or could have simply not considered the possibility of a decline in the housing market, as that was counter to recent history. However, it is also noted that credit agencies draw income from those they rate, which would incentivize a higher rating.

Banks would also benefit from these packages, as they draw less regulatory scrutiny from holding highly rated capital. But in order to create the highly-rated capital (senior tranches), there would necessarily need to be junior tranches (coming from subprime mortgages).

The authors conclude by noting that the rating process has evolved to better capture model and parameter uncertainty (Hello again, Exam 7!), and that it is important to note that certain factors that would be immaterial in the single-asset market are quite prominent in collaterized structures.

Addendum

This isn't in the paper, but it's useful and actually super interesting to have a tiny bit more of background information. In the time surrounding market collapse, federal interest rates were really low, so there was more incentive to look for additional sources of actual interest. This was obviously important to both large investors (banks) and individual investors. This helped fuel the demand for different types of assets, like CDOs. Due to the prior awesomeness of the housing market, it theoretically would not be problematic for a bank to offer a subprime mortgage – in the best case scenario the borrower does not default on the loan, and in the worst case, he does, but the bank gets to keep a house that probably would have increased in value so could easily be sold. Unfortunately however, when the entire market issues mortgages to increasingly credit-unworthy borrowers, a large portion of loans will default and this logic doesn't really work anymore.

When a large portion of the loans are in default, housing prices decline to meet the increased supply. This has a domino effect. People who actually **can** afford to pay their mortgage will be disincentivized to do so, as they realize that they are paying a mortgage that is now worth significantly more than their house. This drives even more sell-offs, and further lowers prices on the assets that were intended to only appreciate in value.

For more about the role of Fannie Mae and Freddie Mac, consider that prior to the existence of these organizations, if a bank wrote a mortgage, it no longer had that money for capital. With a guarantor, the bank could essentially write limitless mortgages, since they could just sell them off shortly thereafter. As expected, this had the (intended) effect of increasing levels of home ownership in the United States. Because these organizations were government-sponsored, they had a significant advantage over other institutions that could potentially purchase mortgages. (They were able to offer a lower interest rate and thereby corner the market.)

Starting around the turn of the century, adjustable-rate mortgages became a big "thing." At this point, interest rates were historically low, and the idea was that the mortgage holder could simply refinance into a fixed rate before the payment adjusted upward. These mortgages were frequently of the subprime variety. Private investors began to purchase these new securities, and Fannie Mae and Freddie Mac saw decreasing market shares, spurring them to guarantee increasing numbers of these types of subprime loans.

Anyway, there were so many reasons leading up to the crash! For more information, check out the video "The 2008 Financial Crisis: Crash Course Economics #12," linked in the image to the right.

Cummins (CAT Bond and Other Risk Linked Securities)

Background

The Cummins article was written in 2008 and was intended at that time to provide information regarding CAT bonds and other reinsurance-type securities in the current market. The market has changed since then, so while the figures he cites are somewhat outdated, you should be aware of the general trends of the market.

Development of the CAT Bond Market

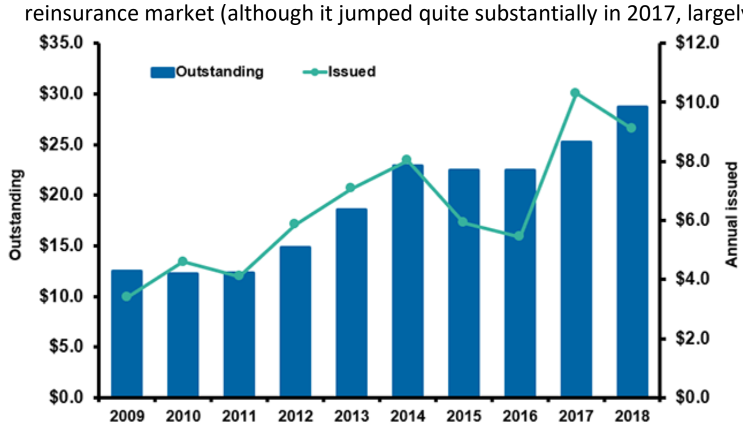
A **catastrophic risk (CAT) bond** is "a fully collateralized instrument that pays off in the occurrence of a defined catastrophic event." CAT bonds provide an interesting and relatively new mechanism of reinsurance for risky perils and are rather significant sources of capital for P&C markets.

In 1992 the Chicago Board of Trade (CBOT) introduced catastrophe futures and derivatives, followed by the Bermuda Commodities Exchange in 1997. Both contracts were withdrawn in short order due to lack of interest in the market. The article cites reasons for disinterest stemming from the thinness of the market, counterparty credit risk, and the lack of desire to break long-standing relationships with reinsurers. **Basis risk** was also of great concern, since the catastrophe structures were not tied to an insurer's specific losses. In 2007, two more exchanges listed CAT contracts, though as of the time the article was written, there was no information as to how those markets would fare.[2]

Another type of security that was introduced in 1995 by Nationwide was the **contingent surplus note**. These were similar to traditional corporate bonds, but included a feature that allowed them to substitute surplus notes with higher rates of return, subject to the discretion of the insurer. Because Nationwide would need to repay the notes, this is not a form of reinsurance, more just a loan. Moreover, it exposes investors to general business risk and potential for default.

CAT bonds are modelled similarly to more traditional asset-backed securities, like mortgage loans and auto loans. CAT bonds pay off on the occurrence of a specified event (e.g., hurricane, earthquake). They

[2] Not super well. The market for CAT bonds is still there, and growing, but it composes only a fraction of the total P&C reinsurance market (although it jumped quite substantially in 2017, largely in reaction to the HIM hurricanes).



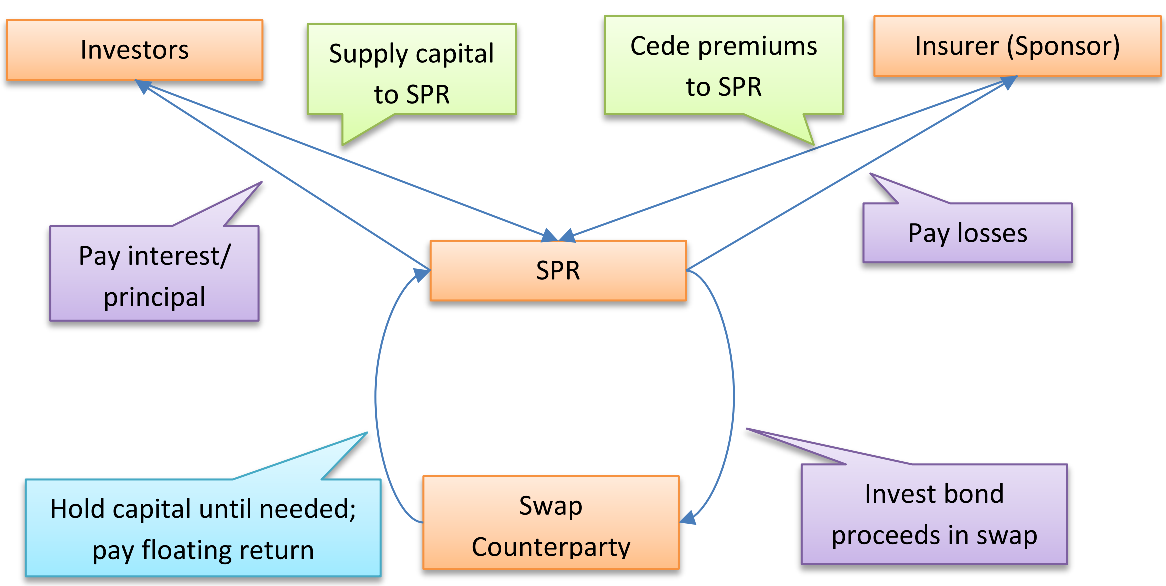
Graph source (https://www.iii.org/fact-statistic/facts-statistics-catastrophe-bonds, Accessed 18 October 2019) are designed to cover risk-remote layers, such as a 1-in-100 year event. These types of events are often not reinsured because insurers would need to concern themselves with the credit risk of the reinsurer, and these levels are often priced with a substantial risk margin. On the other hand, because CAT bonds are fully collateralized by the investors, there is no credit risk. Moreover, they are not very correlated with investment returns, so they may provide lower spreads than would traditional reinsurance.

Another feature of CAT bonds that is advantageous over traditional reinsurance is the multi-year protection. This shields insurers from price changes that may be seen in the reinsurance market, while allowing the bond issuer to spread the fixed costs of issuance over a longer term.

CAT Bond Structure

The article gives an overview of the formation of a CAT bond:

- A single purpose reinsurer (SPR) is formed. The SPR issues bonds to investors and invests in risk-free securities.
  - The bonds include an embedded call feature which allows the SPR to access the entire value of the bond (principal and future interest payments) in the event of a catastrophe. A **principal protected tranche** would guarantee the return of principal even if a catastrophe did take place. These tranches are relatively rare because they would mean less available capital in the event of a catastrophe.
- In return, the bond issuer swaps out the risk-free fixed rate securities for floating rates. This provides protection against changes in inflation.
- Upon termination of the bond contract, assuming no catastrophe occurred, the principal is returned to the bondholder.



The use of an SPR acts as a shield against the risk of an insurer – only in the event of a catastrophe could the investor lose funds, so the investor is not exposed to general business risks. These bonds therefore are attractive to investors who are looking for diversification.

CAT bonds are written based on a stated trigger (which defines the events that will determine the size of payout):

- **Indemnity trigger:** pays out based on the insurer's actual losses.
- **Index trigger:** pays out based on some index, which may include one of the following:
  - **Industry loss indices** are triggered when industry-wide losses exceed a specified threshold.
  - **Modelled loss indices** are based on a specified CAT model's output (EQECAT or Risk Management Solutions are a couple of well-known CAT models) based on a specified geographic area or on the insurer's own exposures.
  - **Parametric indices** are based on specific components of the event, like wind speed of a hurricane.
- **Hybrid trigger:** a mix of the two.

In choosing a trigger, one must consider the tradeoff between moral hazard, transparency, and basis risk. Indemnity triggers ensure the insurer's losses will be covered but require collecting significant amounts of insurer-specific information for pricing. They could cause moral hazard issues from an insurer choosing to generously pay catastrophe losses so that coverage triggers. Index triggers are more transparent, but an insurer's losses could be substantially different from the index measure (basis risk).

The degree of basis risk from an index trigger varies based on several factors. Parametric triggers have low exposures to moral hazard but may have high exposure to basis risk. Industry loss indices on narrowly defined geographic areas have less basis risk than those based on wider areas. Modelled loss indices are subject to model risk, although that is diminishing over time as models improve.

Sidecars

Closely related to CAT bonds, sidecars are SPVs written to provide additional reinsurance capacity. Sidecars are also fully collateralized. The ceding insurer pays premiums upfront, so investors can take advantage of interest on them, though collateral is exposed for the duration of the contract. They are typically off balance-sheet, thus improving leverage.

Catastrophic Equity Puts (Cat-E-Puts)

Cat-E-Puts are another form of contingent financing where the insurer can, in the event of a pre-specified event, issue stock at a pre-agreed upon price in exchange for capital. They are easier to set up than CAT bonds because they do not need to use an SPR. However, because these are not asset-backed securities, the insurer is still exposed to counterparty credit risk.

Catastrophe Risk Swaps

CAT swaps are also not prefunded. They are essentially just a trade of risk between insurance companies. An example of a catastrophe risk swap would be if a company in California traded $50 million of earthquake risk for hurricane risk from an insurer in Florida. The swaps can be designed to expect an equal amount of loss from either side (though this is difficult to do and is subject to inaccuracies). Again, the insurer is exposed to credit risk.

Industry Loss Warranties

Industry Loss Warranties (ILW) are dual-trigger reinsurance mechanisms that have a retention trigger pegged to the incurred losses of the insurer, and a warranty trigger pegged to the losses of the industry. Both triggers need to be hit in order to pay off, but in practice the retention trigger is usually low enough that it's almost guaranteed to be pierced if the warranty trigger hits. ILW are typically one-year contracts and can be written with binary or pro-rata triggers, the latter of which pays off based on how much the loss exceeds the warranty.

Because ILW are pegged to the insurer's actual losses, they receive favorable treatment by regulators (reinsurance accounting), in contrast with regular index-based CAT bonds.

The CAT Bond Market

This section of the article presents a large amount of facts regarding market trends:

- The CAT bond market saw substantial growth in the 10 years leading up to 2007 but still represents only a fraction of the reinsurance market.
- Initially, there were some longer-term (10-year) bonds issued in the 1990s, but the current trend is toward 3-year bonds, which balance a steady stream of income over time with the market participants' lack of desire to be locked into a longer term security.
- The majority (95%) of bond issuers are from the insurance and reinsurance market, with the remainder coming from corporate and government sources.
- The majority of CAT bonds are below investment grade, though this is not tragic since CAT bonds are fully collateralized. The ratings are influenced by the probability that the principal will be eroded by some catastrophe, so the ratings only indicate the layer of catastrophic risk coverage.
- Initially, CAT bondholders and issuers were predominantly insurance and reinsurance companies, though currently (as of 2007), the holders were predominantly CAT and hedge funds. This suggests that CAT bonds are becoming more attractive to investors.
- CAT bonds are typically written with large spreads over LIBOR (investors receive floating interest plus points), but they are still priced well enough to be competitive with traditional reinsurance.

Comparison of CAT bonds and reinsurance

It is somewhat difficult to draw a comparison between pricing of CAT bonds and reinsurance due to lack of data on reinsurance pricing. As a proxy, Guy Carpenter provided data on the relationship between the rate on line $\left(ROL = \frac{\text{reinsurance premium}}{\text{policy limit}}\right)$ and the loss on line $\left(LOL = \frac{\text{expected loss}}{\text{policy limit}}\right)$. They found that CAT bond yields tend to be similar to reinsurance ROL-to-LOL ratios, suggesting they are priced roughly equivalently.

In general, ROL-to-LOL ratios are larger for national insurers than they are for regional insurers. The ratios are lower for contracts with higher expected losses on line, since low LOL contracts cover the riskier upper tails.

Regulatory, Accounting, Tax and Rating Issues

- **Regulatory Issues:** CAT bonds are typically issued offshore (Bermuda, Cayman Islands, Dublin) due to low issuance costs and high levels of expertise in those areas. Onshore transaction costs typically exceed those of offshore and moreover onshore, regulators tend to be concerned about basis risk of nonindemnity bonds, which serves to further impede market development. The authors argue that the contracts could be set up to overstep basis risk, and in no uncertain words condemn the US regulatory structure for being too heavy-handed and rigid.

- **Tax Issues:** Offshore CAT bonds create no problems for sponsors, but onshore bonds are given unfavorable tax treatment. Since the IRS does not explicitly address tax treatment of these bonds, they are taxed as bond dividends rather than interest income. Also, some sponsors treat bond interest in the same way as reinsurance premium.

- **Dissemination of Information on Bonds:** The fact that ~~prospectuses~~ prospecti for privately placed bonds can be distributed only to qualified investors under SEC regulation inhibits opportunities for research on CAT bonds. The authors suggest permitting researchers to access this information in order to improve the robustness of the market.

Other Suggestions

In addition to allowing more access to information on CAT bonds, the authors make other suggestions, including mandating reporting for CAT events once they reach a specified threshold. The additional information would promote market growth. They also suggest deregulation of prices at state level so that insurers would be free to increase prices as necessary to meet loss expectations. As an alternate to deregulation, they suggest giving credit to insurers who lock in multi-year pricing.

Butsic (Solvency Measurement for Property-Liability Risk-Based Capital Applications)

This paper serves to outline how risk can be quantified for purposes of establishing risk-based capital (RBC) for P&C insurers. RBC models are important in determining solvency standards – they aim to quantify how much capital is needed to absorb the risks of insurance.

Economic Basis for Risk-Based Capital

What is capital?

- **Capital** is defined as **Assets – Liabilities**. It represents the owners' stake in the firm.
- In SAP, capital is called **surplus**, and in GAAP, capital is called **equity**.
- A **technical insolvency** results when capital is negative, in which case capital providers lose their stake in the firm, and policyholders take over the assets (generally through liquidation).

Consequences of Insolvency

- The primary parties of insurance contracts are the policyholders and equityholders. Third-party claimants also have vested interest in the insurer's success.
- RBC standards serve as the basis for solvency regulation in the United States.

Desirable Features of a Risk-Based Capital Model

1. Invariant across all classes – the standard should not vary for personal v. commercial insureds, or second- v. third-party claimants, etc.
2. Objectivity – the formula should always point to the same result given the same risk measures, regardless of jurisdiction.
3. Differentiates risk – the formula should have some means of scoring items based on their risk (e.g., an asset portfolio of stocks should be counted differently than a similarly-sized asset portfolio of government bonds.)

Expected Deficit as a Measure of Insolvency Risk

A common measure of risk is the probability of ruin, although this measure has the downside of not considering the severity of ruin.

For example:

	Insurer A	**Insurer B**
Assets	**100,000**	**100,000**
Liability	Uniform from ($0, $125,000)	$\begin{cases} 0; & P = 80\% \\ 312{,}500; & P = 20\% \end{cases}$
Mean Liability	$^1/_2\,(125{,}000) = \mathbf{62,500}$	$80\%(0) + 20\%(312{,}500) = \mathbf{62,500}$
Capital	$100{,}000 - 62{,}500 = \mathbf{37,500}$	**37,500**
Probability of ruin	$1 - \frac{100}{125} = \mathbf{20}\%$	**20%**

The two insurers have very different liability profiles, but under the probability of ruin measure, they appear identical. We can see though that insurer B is clearly worse off.

Insurer's B expected deficit is $80\%(0) + 20\%(312{,}500 - 100{,}000) = \$\mathbf{42,500}$, while Insurer A's expected deficit is substantially less. (This is intuitive – Insurer A's shortfall will never be more than $25,000, the difference between the maximum loss and the total assets. The math about the exact shortfall isn't so important but it's below in case you're curious.[3])

The example demonstrates that the probability of ruin measure is insufficient to really quantify risk differences. A better measure is the **expected policyholder deficit (EPD)**. The **EPD ratio** is the expected deficit as a ratio to expected loss. An example for the case of discrete losses is provided in Table 2 of the text, reproduced in part below.

	Probability	**Asset Amount**	**Loss Amount**	**Deficit**
Scenario 1	0.1	12,000	5,000	0
Scenario 2	0.8	6,000	5,000	0
Scenario 3	0.1	3,000	5,000	2,000

- In this example, the expected assets are 6,300, and expected loss is 5,000, giving 1,300 in capital.
- The deficit is calculated as the shortfall between assets and loss, and is 0 if there is no shortfall.
- The expected policyholder deficit is the probability-weighted average deficit, 200 here.
- EPD ratio = $(\text{Expected Policyholder Deficit}) \div (\text{Expected Loss}) = \frac{200}{5000} = 4\%$

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[3] The expected deficit is the average excess of the liabilities over assets. For Insurer A, there is only a deficit when losses exceed 100,000. The expected shortfall is then:

$$\int_{100{,}000}^{125{,}000} \frac{1}{125{,}000}(x - 100{,}000)dx = \$2{,}500$$

In general, when Assets, A, are fixed and losses are distributed according to $p(x)$, the EPD is defined as:

$$\textbf{Expected Policyholder Deficit (EPD)} = D_L = \int_A^{\infty} (x - A)p(x)dx$$

In this case, the EPD is equivalent to that of a call option with strike price A.

On the other side, if we have certain losses and uncertain assets (which is equivalent to the value of a put option on the ending assets with strike L), we have the following:

$$\textbf{Expected Policyholder Deficit (EPD) — Assets Uncertain} = D_A = \int_0^{L} (L - y)q(y)dy$$

The "uncertain assets/ certain liabilities" EPD formula has rarely shown up, so it doesn't get a cool font.[4] (In insurance we are generally very much concerned with fluctuations of liabilities, so the liability version is much more relevant.)

Setting Capital to a Common EPD Ratio

If regulators use a capital standard such that the EPD ratio is the same for all insurers, we would determine assets needed based on backing out required beginning-of-year assets based on the indicated end-of-year assets needed to satisfy the capital requirement. Using the example from the paper (Table 3):

[4] One student pointed out that this was tested in 2000 (Q44, old Exam 8).

Given the following information about a scenario in which insurers deficit:

	Insurer A	Insurer B	Insurer C
Probability of Deficit	20%	20%	10%
Expected Loss	10,000	10,000	5,000
Beginning Assets	13,000	13,000	6,300
Ending Assets	13,000	13,000	3,000
Size of Deficit	100	5,000	2,000

- The capital is given by Beginning Assets – Expected Loss.
- Suppose the regulators want an EPD ratio of 5%. Then we have $5\% = \frac{\text{EPD}}{\text{Expected Losses}} \rightarrow 5\% E[L] = EPD$
- The deficit within the scenario is (expected deficit) ÷ (probability of deficit)
- The increase/decrease in deficit gives the decrease/increase needed in ending assets.
- We determine BOY assets by scaling the original ratio of EOY Assets to BOY Assets to reflect the current EOY Assets needed.
- Then we can back out required addition to beginning of year capital.[5]

Current Capital	3,000	3,000	1,300
Required Expected Deficit	500	500	250
Deficit in Scenario	$\frac{500}{20\%} = 2{,}500$	$\frac{500}{20\%} = 2{,}500$	$\frac{250}{10\%} = 2{,}500$
Change in Deficit	+2,400	-2,500	+500
EOY Assets Needed	10,600	15,500	2,500
BOY Assets Needed	10,600	15,500	$5{,}250 = 2{,}500\left(\frac{6{,}300}{3{,}000}\right)$
BOY Capital Required	600	5,500	250

Implicit in the work here is that the change in capital won't cause the insurer to default in another scenario. If that were the case, we'd need to perform another iteration of the calculations.

We can also determine EPD ratios on continuous distributions. Supposing we have normally distributed risks, then the formula becomes:

[5] Workup for this table is shown in the Drive (Other Files > ***Excel Supplement***)

EPD Ratio – Normal Distribution of Losses[6]

$$d_L = \frac{D_L}{L} = k\phi\left(-\frac{c}{k}\right) - c\Phi\left(-\frac{c}{k}\right)$$

k = coefficient of variation of losses (standard deviation / mean)

c = ratio of capital to mean loss

$$\phi(x) \ = \ \text{normal density function;} \ \phi(x) = \frac{1}{\sqrt{2\pi}} e^{-(x^2/2)}$$

$\Phi(x)$ = cumulative normal distribution function, pulled from table

The equivalent for the uncertain asset scenario is:

$$d_A = \frac{D_A}{L} = \frac{1}{1 - c_A}\left[k_A\phi\left(-\frac{c_A}{k_A}\right) - c_A\Phi\left(-\frac{c_A}{k_A}\right)\right]$$

Note that the formulas above use the lower-case d to represent the ratio of expected policyholder deficit to losses. This adjustment allows for a ready comparison of risk elements of different sizes.

I've not seen this appear on any past exams (though it's easy enough to memorize given how similar it is to the uncertain liabilities scenario), but there are some conclusions that can be drawn when comparing the formulas for asset and liability ratios:

- For the same coefficient of variation, for d_A to equal d_L, more capital (relative to assets) is required than for losses (relative to losses).
- Asset risk requires more capital under the normal distribution than does loss risk because if assets and losses have the same coefficient of variation, the dollar standard deviation of assets is larger (since assets exceed liabilities). Thus, the capital required has a larger standard deviation.

[6] If you're interested, the derivation of the normal and lognormal EPD formulas are provided in the appendix of the reading, and they are based off the idea that EPD can be valued as a financial option.

Using the following set of examples, assuming a normal distribution, we note that the capital required to produce the same EPD ratio under the unknown asset portion is relatively larger, although the coefficient of variation is the same as under the uncertain losses scenario.

	Fixed Assets, Losses Uncertain	**Fixed Losses, Assets Uncertain**
Assumptions	Mean Loss = 1,000 St. Dev. of Losses = 250 Capital = 500 ∴ Assets = 1000 + 500 = 1,500	Mean Assets = 1,500 St. Dev of Assets = 375 Capital = 865.5
CV	$k = 250 \div 1000 = 0.25$	$k_A = 375 \div 1500 = 0.25$
Capital ratio	$c = 500 \div 1000 = \mathbf{0.5}$	$c_A = 865.5 \div 1500 = \mathbf{0.577}$
$\boldsymbol{\phi}$	$\phi(-0.5/0.25) = 1/\sqrt{2\pi}\, e^{-2^2 \div 2} = 0.054$	$1/\sqrt{2\pi}\, e^{-(0.577/0.25)^2 \div 2} = 0.0278$
Φ (probability of ruin)	$\Phi(-2) = 0.0228$	$\Phi\left(-\frac{0.577}{0.25}\right) = \Phi(-2.308) = 0.0105$
EPD Ratio	$(0.25)(0.054) - (0.5)(0.0228) = \mathbf{0.21\%}$	$\frac{1}{1-0.577}\begin{pmatrix}0.25 \cdot 0.0278 - \\ 0.577 \cdot 0.0105\end{pmatrix} = \mathbf{0.21\%}$

I bet you are thinking that the normal distribution is frequently shown to be inappropriate for modelling most insurance losses, particularly when losses do not occur independently, and are wondering why the formulation assuming a lognormal distribution is not provided instead.

Well, *good news everyone*, the reading does also provide the lognormal formulation**:**

EPD Ratio – Lognormal Distribution of Losses

$$\boldsymbol{d_L = \Phi(a) - (1 + c)\Phi(a - k)}$$

k = coefficient of variation of losses (standard deviation / mean)

c = ratio of capital to mean loss

$$a = \frac{k}{2} - \frac{\ln(1+c)}{k}$$

$\Phi(x)$ = cumulative normal distribution function, pulled from table

The equivalent formulation for uncertain assets which are lognormally distributed is:

$$d_A = \Phi(b) - \frac{\Phi(b - k_A)}{1 - c_A}; \quad b = \frac{k_A}{2} + \frac{\ln(1 - c_A)}{k_A}$$

Butsic notes that advantages of the lognormal distribution are that negative values are impossible and it more appropriately reflects the skewness of losses.

Comparing the two presented distributions, note:

- Unlike the normal distribution, under the lognormal distribution, when assets and losses have the same coefficient of variation, $d_A = d_L$ with a smaller capital ratio for assets than for losses (due to asymmetry of the lognormal function, plus the fact that while losses are uncapped, assets are floored at 0).
- The capital requirement for losses under the lognormal distribution is larger than under the normal distribution, especially as the coefficient of variation increases.

Looking at another example where this time we assume that uncertain quantities follow a lognormal distribution, we observe that given the same coefficient of variation, a smaller capital:asset ratio produces the same (-ish) EPD ratio.

	Fixed Assets, Losses Uncertain	**Fixed Losses, Assets Uncertain**
Assumptions	Mean Loss = 1,000 St. Dev. of Losses = 250 Capital = 500	Mean Assets = 1,500 St. Dev of Assets = 375 Capital = 500
CV	$k = 250 \div 1000 = 0.25$	$k_A = 375 \div 1500 = 0.25$
Capital ratio	$c = 500 \div 1000 = \mathbf{0.5}$	$c_A = 500 \div 1500 = \mathbf{0.333}$
$\boldsymbol{a}, \boldsymbol{b}$	$a = \frac{0.25}{2} - \frac{\ln(1+0.5)}{0.25} = -1.497$	$b = \frac{0.25}{2} + \frac{\ln(1-0.333)}{0.25} = -1.495$
EPD Ratio	$\Phi(-1.497) - (1+0.5)\Phi(-1.497-0.25) = \mathbf{0.675}\%$	$\Phi(-1.495) - \frac{\Phi(-1.495 - 0.25)}{1-0.333} = \mathbf{0.672}\%$

Additional Considerations in Measuring Risk

Accounting Conventions

When probability distributions are selected for liabilities or assets, the shape of the distribution will depend on the forecast time. Accounting conventions can present a difficulty with this since their represented values may change even when the risk does not. Butsic suggests that market-value accounting is the most appropriate method for assessing solvency, since it uses current realizable value for its balance items. (One would still need to remove the value of intangible assets and goodwill when using market valuations.)

The accounting book value hinders the use of RBC methodologies since the recorded values generally do not align with actual realizable values. Additionally, the use of accounting book values would not meet the desirable quality of invariance, since both SAP and GAAP allow identical items to be recorded at different amounts (the paper cites as an example including a margin in loss reserves versus discounting to present value). Thus, when using financial statements for setting risk-based capital, the user must first remove the accounting biases.

Time Horizon

In addition to accounting methodology, time also plays a significant role in determining RBC levels: the potential for significant deviation from expectations is much larger over a 5-year period than it is over a 6-month period. It is therefore necessary to establish a common time horizon when valuing different risk elements.

To allow for consistency in measurement, Butsic recommends ratios of items on common financial statements. Capital ratios measure capital to liabilities, items which both appear on the balance sheet, so represent a reasonable choice for ratio. A leverage ratio like premium to surplus would be a less-than-ideal choice since premium appears on the income statement and surplus on the balance sheet. Other reasonable ratios include reserves to surplus or incurred loss to premiums.

Example (Table 4 from paper) [7]

Suppose we were comparing assets (stocks) to reserves. We are given that the value of stocks at the end of four years, and the value of unpaid loss at the end of one year both have standard deviation of 0.1 times the current value. Additionally, both have normally-distributed diffusion processes. The company's beginning capital = 100

Then the EPD ratios for each would be given by:

	Value	Standard Deviation[1]		Probability of Ruin[2]		EPD[2]	
	Now	**1 Year**	**4 Years**	**1 Year**	**4 Years**	**1 Year**	**4 Years**
Stock	1,000	50	100	0.0228	0.1587	0.425	8.332
Reserves	1,000	100	200	0.1587	0.3085	8.327	39.563

[7] Workup for this table is shown in the Drive (Other Files > ***Excel Supplement***)

Notes

1. Standard deviation is given as 100 for stocks after 4 years and reserves after 1 year. For stocks, standard deviation for one year is solved by $4\sigma_1^2 = (100)^2 \rightarrow \sigma_1 = 50$. For reserves, standard deviation for four years is solved by $4(100)^2 = \sigma_4^2 \rightarrow \sigma_4 = 200$
2. For reserves, use $d_L = k\phi\left(-\frac{c}{k}\right) - c\Phi\left(-\frac{c}{k}\right)$;
   - Reserves, 4 years: $c = \frac{100}{1{,}000} = 0.1; k = \frac{200}{1{,}000} = 0.2$

$$d_L = 0.2\phi(-0.5) - 0.1\Phi(-0.5) = 0.2 \cdot \frac{1}{\sqrt{2\pi}} e^{-(0.5^2/2)} - 0.1(0.3085) = 0.03956$$

$$D_L = .03956 \times 1{,}000 = \mathbf{39.56}$$

$\mathbf{\Phi(-0.5)}$ **= 0.3085** using a normal table; this is probability of ruin.

   - Reserves, 1 year is the same as above except $k = 0.1$

For stocks, use $d_A = \frac{1}{1-c_A}\left[k_A\phi\left(-\frac{c_A}{k_A}\right) - c_A\Phi\left(-\frac{c_A}{k_A}\right)\right]$

   - Stocks, 1 year: $c_A = \frac{100}{1{,}000} = 0.1; k_A = \frac{50}{1{,}000} = 0.05$

$$d_A = \frac{1}{1-0.1}\left[0.05\phi\left(-\frac{0.1}{0.05}\right) - 0.1\Phi\left(-\frac{0.1}{0.05}\right)\right]$$

$$= \frac{1}{0.9}\left[0.05 \cdot \frac{1}{\sqrt{2\pi}} e^{-(2^2/2)} - 0.1(0.0228)\right] = 0.000466$$

$$D_A = 900 \times 0.000466 = 0.452$$

Note: Butsic uses liabilities of 1,000 here, so the original table in Butsic does not match the stock EPD values shown. Liabilities of 1,000 does not appear consistent with the assumption of 100 in capital and 1,000 in stock assets.

$\mathbf{\Phi\left(-\frac{0.1}{0.05}\right) = 0.0228}$ using a normal table; this is probability of ruin.

Stocks, 4 years is the same as above except $k_A = 0.1$

Insurer as a Going Concern

The above discussions made no consideration for risk from new and renewal business, only the runoff. As future policies present a large uncertainty for insurers, they should also be incorporated into risk models. This will cause end-of-year capital to be influenced not only by the runoff of beginning liabilities, but by the value of business added during the time period.

The end-of-period assets and liabilities could then be denoted as:

$$A_1 = (A_0 + P)(1 + r)$$

$$L_1 = L_0(1 + g) + L_P$$

- A_0: beginning-of-period assets
- P: premium (net of expenses)
- r: return on assets (a random variable, asset risk)
- L_0: initial liabilities
- g: rate of change in the value of liabilities (a random variable; reserve risk)
- L_P: additional liabilities incurred from new business written during the period

With this consideration, we can consider the EPD to be a function of asset risk, reserve risk, and risk from losses on new business. Because each of these risks is a balance sheet item, the capital adjustment process guarantees a minimum EPD for policyholders, even if more exposures are written during the period.

Insolvency Cost as a Financial Option

We note that the EPD ratio developed thus far is based on the end-of-year value of liabilities and assets. If we consider the present value of the EPD, we would multiply by a factor of $1/(1 + i)$ to account for the time value of money. Then the value of the EPD on a liability risk element paired with a riskless asset is equivalent to that of a call option with an exercise price equal to the value of end-of-year assets.

Example: Suppose that a liability with current value $1,000 has a 50/50 chance of having value either $1,200 or $800 at the end of the year. End of year assets are known at $1,100.

EPD at end of year = $\$100(50\%) = \$50$

EPD at beginning of year (using 8% interest) = $\$50 \div 1.08 = \$\mathbf{46.30}$

Now suppose we have a stock with current value $1,000. It will be worth either $1,200 or $800 at the end of the year. Consider a 1-year European call option with strike price $1,100. (A call option allows us to purchase the stock for $1,100.)

If the stock price increases to $1,200, the option is worth $100 at the end of the year. If the price drops, the option is worth $0, since it will not be exercised. Therefore, the value of the call today is $100(50%)/1.08 = **$46.30**, the same as the EPD.

By the same reasoning, an asset risk with a riskless liability has an EPD which is the equivalent to the value of a put option on the ending assets, with strike price equal to the value of the liability in one year.

We could also formulate an EPD on the risky asset/ risky liability situation as a put option with strike price zero.

Correlation of Risk Elements

A final consideration is the degree of correlation of assets and liabilities. As discussed in other parts of the syllabus, misestimation of the extent of correlation has a significant impact on the indicated level of capital required.

Example (Table 10 from paper)

Suppose that we have two lines of business, each with end-of-year assets fixed at $6,900, and end-of-year losses equal to 2,000 with probability 60%, or 7,000 with probability 40%.

Perfectly Correlated

If the lines are perfectly correlated, then end-of-year losses for the combined lines are 4,000 with probability 60%, and 14,000 with probability 40%. Assets are fixed at $13,800, so the EPD is 40%($200) = **$80.**

Independent

If the lines are independent, then end-of-year losses have the following distribution:

	Line 2 = 60%	Line 2 = 40%
Line 1 = 60%	36%	24%
Line 1 = 40%	24%	16%

	Line 2 = 2,000	Line 2 = 7,000
Line 1 = 2,000	4,000	9,000
Line 1 = 7,000	9,000	14,000

$$\text{Loss} = \begin{cases} 4{,}000 & 36\% \\ 9{,}000 & 48\% \\ 14{,}000 & 16\% \end{cases}$$

Default occurs only in the third scenario, in the amount of $200. The EPD is 16%(200) = **$32**

Wow, that's a pretty big drop in EPD!

Continuous Distributions

We could also look at the case where our variables are not discrete, but continuous normal. Then mean required capital can still be expressed as the difference between expected assets and liabilities, and variance is a function of the correlation between the two variables, as we saw in BKM:

Mean and Variance of Capital with Two Normally Distributed Assets or Liabilities

Capital = Assets – Liabilities

Random Variable	Mean	Variance of Capital
Two Assets (Liabilities fixed)	$C = (A_1 + A_2) - L$	$\sigma_C^2 = \sigma_1^2 + \sigma_2^2 + 2\rho\sigma_1\sigma_2$
Two Liabilities (Assets fixed)	$C = A - (L_1 + L_2)$	
Asset and Liability	$C = A - L$	$\sigma_C^2 = \sigma_A^2 + \sigma_L^2 - 2\rho\sigma_A\sigma_L$

Note: The negative variance adjustment for the asset/liability case is based on the variance of the difference of two correlated random variables:

$$Var[X \pm Y] = Var[X] + Var[Y] \pm 2\sigma_{XY}$$

Since capital is assets minus liabilities, the variance of capital when we are dealing with both random assets and liabilities will take a negative adjustment.

Some special cases based on the above:

	Two Assets or Two Liabilities	Asset and Liability
Perfect positive correlation $\boldsymbol{\rho = 1}$	$\sigma_C^2 = \sigma_1^2 + \sigma_2^2 + 2\sigma_1\sigma_2 = (\sigma_1 + \sigma_2)^2$ $\boldsymbol{\sigma_C = \sigma_1 + \sigma_2}$	$\sigma_C^2 = \sigma_A^2 + \sigma_L^2 - 2\sigma_A\sigma_L = (\sigma_A - \sigma_L)^2$ $\boldsymbol{\sigma_C = \sigma_A - \sigma_L}$
Perfect negative correlation $\boldsymbol{\rho = -1}$	$\boldsymbol{\sigma_C = \sigma_1 - \sigma_2}$	$\boldsymbol{\sigma_C = \sigma_A + \sigma_L}$
Independent $\boldsymbol{\rho = 0}$: *Square root rule*	$\sigma_C^2 = \sigma_1^2 + \sigma_2^2$ $\boldsymbol{\sigma_C = \sqrt{\sigma_1^2 + \sigma_2^2}}$	$\sigma_C^2 = \sigma_A^2 + \sigma_L^2$ $\boldsymbol{\sigma_C = \sqrt{\sigma_A^2 + \sigma_L^2}}$

The EPD ratio for a combination of normally-distributed risky elements can be determined in the same way seen earlier for normally distributed assets or liabilities.

Likewise, the EPD ratio for a combination of lognormally-distributed elements will use the formulas seen earlier.

<table>
<tr><th>EPD Ratio – Normal Distribution of Losses</th><th>EPD Ratio – Lognormal Distribution of Losses</th></tr>
<tr><td>$$d_L = \frac{D_L}{L} = k\phi\left(-\frac{c}{k}\right) - c\Phi\left(-\frac{c}{k}\right)$$ $\phi(x)$ = normal density function; $\phi(x) = \frac{1}{\sqrt{2\pi}}e^{-(x^2/2)}$</td><td>$$d_L = \Phi(a) - (1+c)\Phi(a-k)$$ $$a = \frac{k}{2} - \frac{\ln(1+c)}{k}$$</td></tr>
<tr><td colspan="2">k = coefficient of variation of losses (standard deviation / mean)
c = ratio of capital to mean loss
$\Phi(x)$ = cumulative normal distribution function, pulled from table</td></tr>
</table>

For both the normal and lognormal distribution, the relationship between c and k is approximately linear when d and k are both small. Then, **the required capital for the EPD standard is approximately proportional to the standard deviation of the risk**.

Example (Table 13 from paper)

Below we have risk elements from a hypothetical insurer's balance sheet. The capital ratios are based on a 0.005 EPD ratio.

All elements are assumed to be lognormally distributed.

Risk Element	Amount	Capital	
Stocks	200	40	Assets
Bonds	1,000	50	
Affiliate Stocks	1000	20	
Loss Reserve	800	320	Liabilities
Property UPR	100	20	
Total		**450**	The sum of 450 here assumes that *all* items are fully correlated.

Correlation coefficients:

	Bonds	Affiliates
Stocks	0.2	1.0
Bonds		0.2
Loss Reserve	0.3	-1.0

Generalizing the continuous distribution capital requirements for sums of risky items above, we can adjust the required capital to be approximately:

- Capital amounts: $40^2 + 50^2 + 20^2 + 320^2 + 20^2 = 107{,}300$
- Correlation adjustment (positive for items on same side of balance sheet, negative for items on opposite sides):

  $(0.2)(40)(50) + 1.0(40)(20) + 0.2(50)(20)$

  $\underline{-0.3(320)(50) - (-1.0)(320)(20)}$

  $= 3{,}000$
- **Capital =** $\boldsymbol{\sqrt{107,300 + 2 \times 3,000} = 337,}$ much less than the 450 above.

If we consider only the impact of the bond/reserve correlation, by setting it equal to 0, the required capital would be 351. Therefore, that correlation effect was a reduction in required capital by 14.

Anyway, the whole point is that correlation has a big effect on capital calculations.

Goldfarb (Risk-Adjusted Performance Measurement for P&C Insurers)

This paper is a doozy! It covers many ideas, very densely, so this one would be a good one to read carefully. I've specifically tried to summarize this paper in outline form, while keeping the big idea in mind, so as you read the summary, try to keep in mind how all the individual pieces fit within the broad purpose of the paper, which is to delineate the process for determining risk-adjusted capital.

Risk-Adjusted Performance Measurement

Risk-adjusted measures are used by firms for capital planning, risk management, and corporate strategy decisions. The basic form of return on capital is given by taking the ratio of income to capital. Some common measures include return on equity (ROE), return on assets (ROA), and total shareholder return (TSR). The shortcoming common to all of these measures is that they fail to incorporate differences arising from activities with varying degrees of risk or uncertainty. The **risk-adjusted return on capital (RAROC)** measure looks to improve upon the basic form by requiring more income for riskier lines of business. RAROC then, requires two components – income and capital.

RAROC Numerator: Income

Income can be measured in a variety of ways, four of which are noted in the paper.

1. **GAAP Net Income**: This is a measure based on GAAP accounting standards, appropriate when guiding management decisions.
2. **Statutory Net Income**: This measure is based on STAT accounting.
3. **IASB Fair Value Basis Net Income:** This income measure is derived from the International Accounting Standards Board and is intended to be a measure of "fair value." Key differences between this measure and US GAAP are that the IASB uses discounting of loss reserves as well as a risk margin in the valuation of liabilities.
4. **Economic Profit**: Economic profit is intended to remove most accounting biases. In this measure, assets are valued at their market value, and liabilities are discounted to present value, and may or may not include a provision for risk. This represents an improvement over GAAP or STAT valuation, but includes some limitations:
    - Economic profit does not include the franchise value of a firm (the value of future profits), a key source of the overall value of the firm. (See Panning.)
    - The use of economic profit complicates reconciliation to GAAP, which is likely important to management and other users of the firm's results (such as investors, regulators, and rating agencies).

RAROC Denominator: Capital

Capital may be risk-adjusted, as used in RAROC, or not risk-adjusted, as used in ROC.

The **non-risk-adjusted measures of capital** include:

1. **Actual Committed Capital:** Cash provided to company by shareholders. This is contributed capital, plus retained earnings, based on GAAP, STAT, or IASB.
2. **Market Value of Equity:** This includes actual committed capital, adjusted for market value of assets and liabilities, *as well as* the franchise value of the firm.

The **risk-adjusted measures of capital** include:

1. **Regulatory Required Capital**: such as that required by an RBC model.
2. **Rating Agency Required Capital:** This is capital required to achieve or maintain a certain credit rating, as determined by rating agencies' capital models.
3. **Economic Capital:** The capital required to achieve a specified goal, over a specified time horizon, with a specified degree of probability. For example, the firm may desire to meet a *solvency objective*, which would ensure its ability to meet obligations to current policyholders. Alternatively, the firm may desire to meet a *capital adequacy objective*, whereby it would look to ensure it has enough capital to pay dividends and support growth.

   Economic capital can be interpreted in a variety of ways. Under STAT accounting, it can be considered as the difference between the financial resources needed and the *undiscounted value* of liabilities. Alternately, it could be the excess of financial resources above the *discounted* liabilities. Still another method would take the excess over the *fair value* of liabilities, including a risk margin.
4. **Risk Capital:** Instead of using economic capital, which may be interpreted in many ways, one might choose to use risk capital, the amount that shareholders must contribute to absorb the risk not incorporated in reserves or in premiums. This measure may be identical to economic capital if economic capital is defined to include a risk margin (or if no margin is included in premiums or reserves).

$$\text{Risk Capital} = \text{Potential Liabilities} - (\text{Premiums} + \text{Reserves})$$

The steps used in the paper to determine the amount of capital assigned are as follows, the first of which has been described above.

Risk-Based Capital Caveats

Risk-based measures of capital can be much lower than either the firm's book value or market value of equity. As a result, if we attempt to reflect the cost of capital allocated to a given line, the true costs might be understated due to stranded capital. Some practitioners will adjust the return measures to reflect the cost of the actual capital held in excess of risk-based capital.

Determining Capital to Guide Strategic Decisions
*Define Capital > **Measure Risk** > Set Risk Threshold > Analyze Risk Sources > Aggregate Risk > Determine Capital by Line of Business*

Risk Measures

Once the firm has chosen a definition of capital, it must then determine how to measure it. Some such measures are:

1. **Probability of Ruin**: This most often is the same measure as probability of default, but could also be more generally applied, such as probability of a ratings downgrade.

2. **Percentile Risk Measure (Value-at-Risk)**: The fact that this has come up in yet another paper should be a really good indication that it's pretty commonly tested on the exam! Recall that V@R at a given level α is the amount of capital that provides for a $1-\alpha$ probability of losses exceeding that level.

   Goldfarb differentiates between Percentile Risk Measure and V@R with some subtle nuances. Because the quantity modelled is not necessarily the value of cash flows (since they generally do not include margin and discounting), it's not really a *value* per se, which is why he indicates the term "percentile risk measure" is more accurate.

3. **Conditional Tail Expectation**: The CTE or tail value-at-risk (TV@R) is the linear average expected loss of those scenarios where loss exceeds some specified threshold (i.e., those losses above the $1-\alpha$ level of probability).

4. **Expected Policyholder Deficit Ratio**: Expected policyholder deficit (EPD) is similar to CTE, where the chosen threshold is that where liabilities exceed assets. The policyholder deficit is zero when there is no shortfall between liabilities and assets, and equal to (Liabilities – Assets) in those scenarios where there is a shortfall. The EPD ratio is the total policyholder deficit divided by expected losses. The Butsic paper from the syllabus develops formulas that can be used when risks follow normal or lognormal distributions.

We demonstrate the calculations of each of these risk measures using the below sample data of 1,000 sample losses, where the mean loss is 13,000 and the firm is currently holding assets of 21,000.

Scenario	Loss	Shortfall
1000	176,200	(155,200)
999	120,100	(99,100)
998	110,300	(89,300)
997	109,000	(88,000)
996	107,100	(86,100)
995	100,200	(79,200)
994	89,800	(68,800)
993	89,600	(68,600)
992	86,200	(65,200)
991	86,100	(65,100)
990	82,400	(61,400)
989	81,800	(60,800)
…		
849	22,000	(1,000)
848	21,700	(700)
847	21,700	(700)
846	21,700	(700)
…		
840	21,300	(300)
839	21,200	(200)
838	21,000	-
…		
6	700	-
5	600	-
4	600	-
3	500	-
2	500	-
1	300	-

Probability of Ruin: If we define ruin to be default, then in scenarios 839 forward, the firm is ruined. Probability of ruin would be $1 - \frac{838}{1000} = \mathbf{16.2}\%$

Note that this is kind of a backwards setup – typically, the company would want to determine the capital required to keep probability of ruin below a certain threshold – in this case we are determining the probability of ruin, given the assets. V@R is a more typical application of probability of ruin.

Value-at-Risk: If we require capital at the V@R(99) level, we would need enough to be covered in all but the worst 1% (or the 10 worst) scenarios. We would need $82,400 to cover the 10th worst loss, so the firm would need to increase assets to that level. (In past exams, the amount indicated by either the 991st or 990th worst loss would usually be acceptable.)

Conditional Tail Expectation: Using a 99% level again, we take the average of losses from scenarios 991 – 1000, getting $107,460 in required assets.

Expected Policyholder Debt Ratio: For this we use the shortfall column, which is the difference between assets and losses when losses exceed assets and 0 otherwise. Here, the average shortfall is 2,886, and the EPD ratio is $\frac{2,886}{13,000} = 22.2\%$

You can note that all four of these risk measures point to the company probably wanting to hold more capital than it currently has.

Determining Capital to Guide Strategic Decisions
*Define Capital > Measure Risk > **Set Risk Threshold >** Analyze Risk Sources > Aggregate Risk > Determine Capital by Line of Business*

Risk Measurement Threshold

Once a risk measure is chosen, the company needs to identify the threshold at which to measure the risk (e.g., what default probability, what percentile level for V@R or TV@R, etc.). To determine an acceptable target, the company may consider a variety of methods:

1. **Bond default probability at selected credit rating level:** The firm could look at the probability of default for a specific bond level, such as an AA-rated security. At the very least, the firm would clearly need to select which rating to target. Additionally, it would need to distinguish between being placed into run-off or being downgraded. Typically, models do not measure ability to retain a rating with some specified probability; rather they assume run-off and measure ability to withstand a tail event (this disregards franchise value).

   Considering that an AA-rated bond has a 1-year default probability of 0.03%, firms should be wary of extrapolating information on such a remote possibility (modelling difficulties), and need to consider whether the estimates of default rates should be based on historical estimates (for stability) or current estimates, so as to better reflect current market conditions.

   Should historical default statistics be used, the firm would need to consider which source is appropriate (as not all rating agencies provide the same data). Finally, the firm should consider what time horizon is appropriate – rating agencies typically quote a 1-year probability of default, which may not be appropriate for all lines of business.

2. **Management's Risk Preferences:** A relevant measure of the appropriate amount of risk tolerance would be one that reflects management's preferences. This can differ based on perspective. For publicly-traded companies, shareholder opinion is also relevant, and investors would probably not be overly concerned with probability of default, although policyholders would.

3. **Arbitrary Default Probability, Percentile, or EPD Ratio:** The firm could randomly select a percentile that seems high enough to be tenable.

Determining Capital to Guide Strategic Decisions
Define Capital > Measure Risk > Set Risk Threshold > ***Analyze Risk Sources*** *> Aggregate Risk > Determine Capital by Line of Business*

Risk Sources

Once the firm has selected a risk measure as well as the corresponding threshold for that measure, their next step is to consider the sources of material risk to which the firm is exposed. Typical risk categories include:

1. **Market Risk:** Market risk captures changes in current investments from changes in equity indices, interest rates, and foreign exchange rates. The distribution of future portfolio values can be estimated, and then V@R or TV@R risk measures are applied.

   Selecting an appropriate time horizon can be tricky – generally the horizon could be 10 or fewer days, which is the time it would take to liquidate such assets, but insurance companies would be concerned with longer time periods (for consistency with runout on their lines of business), introducing substantial estimation error. For simplification, one may ignore the discrepancy in horizons.

2. **Credit Risk:** Credit risk captures loss in value due to counterparty default or change in counterparty rating or certain yield spreads. We can include in this category:

   - marketable securities, derivatives, and swap positions
   - contingent premiums and deductibles
   - reinsurance recoveries

3. **Insurance Risk:** Insurance risk includes loss reserves from prior policy years, underwriting from the current policy year, and property catastrophe risk.

   3a. **Loss reserve risk** comprises three components – *process risk* (the risk that actual losses differ from expected due to the inherent variability of insurance), *parameter risk* (the risk that actual losses differ from expected due to inability to correctly estimate model parameters), and *model risk* (the risk that actual losses differ from expected due to use of incorrect model to estimate expected). In practice, sometimes model risk is considered to be a type of parameter risk.

   Reserve estimation error can be used to quantify the uncertainty contained within a reserve estimate, often in the form of a confidence interval. Alternately, a full reserve distribution can be provided to depict the full range of possible losses. To determine a full distribution, there are several commonly-used methods, many of which you might remember from Exam 7:

   i. Mack Method: provides an estimate of standard error using a traditional volume-weighted chain ladder approach. This was covered exhaustively in Exam 7, and to some extent in a short example in this paper, but I will not delve into details here.

   ii. Hodes, Feldblum, Blumsohn: Simulates age-to-age loss development factors to use in chain ladder. This is processing-time heavy, and simulation may not be reliable.

iii. Bootstrapping: Simulates hypothetical loss triangles based on distribution of paid or incurred loss amounts. The distribution of ultimate loss amounts can be derived using a large number of simulations.

iv. Zehnwirth Methods: Uses log of incremental paids and identifies trends simultaneously affecting accident years, calendar years, and development periods.

v. Panning Econometric Approach: Intended to improve upon certain assumptions used in the traditional chain ladder approach. Relies on linear regression techniques to minimize squared errors, uses incremental data (to avoid serial correlation introduced by cumulative triangles), and models each development period separately (to account for heteroscedasticity).

3b. **Underwriting risk** captures deviations arising from current or future new business. For new business, there are several methods to attempt to quantify risk, many of which are closely related to topics you might remember from your first big boy exam.

- Loss Ratio Distribution Models: Here we can simply use a distribution of loss ratios in conjunction with an estimate of written premium over the desired horizon. The distributions can be determined based on historical loss ratio data (adjusted for trends, premium adequacy, and volume) or on industry data (or both), and one must also consider what is an appropriate model for estimating the distribution of the loss ratio. Some common choices are normal, lognormal, and gamma.

- Frequency & Severity Models: Collective risk models, which model frequency and severity separately, can be implemented, and may provide more robust models than do loss ratio models due to the use of more data. In addition, this method may be preferable because it more easily accounts for growth in volume of business, inflation, changes in limits and deductibles, impacts of deductibles on claims frequency, and allows for consistent estimates of loss shares between insured, insurer, and reinsurer.

  After modelling claim frequency and severity, the aggregate loss distribution can then be determined by utilizing a closed form solution, numerical methods, simulation, or approximations using fitting to moments. The paper describes this last approach in a small amount of detail, but the derivations should already be familiar to you from prelims. Simulation has the advantage of allowing simple modelling of complex policy structures, but the drawback of huge processing times.

3c. **Property Catastrophe Risk**: Historically, catastrophe risk “models” were not a very robustly developed concept. Instead, insurers principally used estimates from historical experience, combined with a large amount of hope. More recently developed catastrophe risk models utilize meteorological, seismological, and engineering data to create a probability distribution of expected losses. These distributions can be compared with estimations for property damage to determine impact of catastrophe events.

The modules of catastrophe models may be familiar from Exam 8 and include a hazard module to quantify the potential events, a vulnerability module used to measure the extent of damage from an event, and a financial analysis module to determine monetary impact to insurer and/or reinsurer.

4. **Other Risk Sources:** There are innumerable other possible sources of risk, including various kinds of operational and strategic risks. They tend to be less amenable to quantification and modelling.

Determining Capital to Guide Strategic Decisions
Define Capital > Measure Risk > Set Risk Threshold > Analyze Risk Sources > ***Aggregate Risk*** *> Determine Capital by Line of Business*

Risk Aggregation

Now that we've captured "all" sources of risk, we can move forward with determining an aggregate distribution for all the risk sources (implicitly assuming all risk measures were quantified under the same horizon). Clearly, we cannot just simply sum up all the expected distributions due to the sizeable impact of correlation.

Goldfarb distinguishes between correlation and dependency by noting that correlation refers to a specific measure of linear dependency, and so dependency is the more general term which he utilizes for the remainder of the paper.[8]

So, how can one determine the level of dependency across (or within) risk categories? The paper outlines three common methods:

1. **Empirical Analysis of Historical Data**: This is intuitively appealing, but suffers from impracticality due to lack of historical data and a tendency to produce unreliable and inconsistent measures of dependency even when data issues are not a problem. Further, while historical data can certainly be helpful to determine correlation during "regular" events, the estimates may be wildly inappropriate when viewed in the context of tail events.

2. **Subjective Estimates:** This method, which reflects the user's opinion and intuition about dependency, is also intuitive as it would necessarily lead to results that are in line with the user's expectations. The most obvious drawback is common to any subjective measure – lack of structure. Additionally, as the sources of risk and affected lines of business grow, the number of estimates required grows faster.

3. **Explicit Factor Models:** In this approach, we link the variability of assets to common factors, and then correlations can be derived based on each asset's respective sensitivity to these factors. Explicit factor models can be used to reflect dependency across lines of business, and across the reserve and underwriting risk categories, but would require separate assumptions to reflect dependency across the other risk categories.

[8] In practice, the two are used interchangeably (or correlation is perhaps used more frequently).

After determining dependency measures, we can create an aggregate risk distribution using one of a variety of measures.

1. **Closed Form Solutions:** These are better used as exemplars to demonstrate how aggregate risk distributions are formed – in practice, the wide variety of individual risk distributions can make these solutions cumbersome.
2. **Approximation Methods:** Instead of deriving analytical formulas, aggregate distributions can be simulated, usually using some sort of simplifying assumption (such as assuming all distributions are lognormal).
3. **Simulation Methods**: Simulation is a nice way to visually capture a distribution, albeit rather process-heavy. Dependencies can be reflected using the Iman-Conover Method (which uses a rank correlation measure to separately simulate each variable and then re-shuffles results to preserve the rank correlation) or with copulas (multivariate distributions that can be selected based on the desired level of dependency at given percentiles). Both of these methods are discussed a bit in Exam 7.

If the practitioner does not wish to model the aggregate risk distribution to the end of calculating an aggregate risk measure, he may simplify the process of calculating the aggregate risk measure through the commonly used square root rule, which calculates the aggregate risk measure C as:

Square Root Rule for Aggregate Risk Measure

$$C = \sqrt{\sum C_i^2 + \sum_i \sum_{j \neq i} \rho_{ij} C_i C_j}$$

This measure is exact when each risk distribution is normal and when the risk measure is proportional to standard deviation (as in the relative V@R measure), but otherwise is somewhat of a crude approximation.

Determining Capital to Guide Strategic Decisions
Define Capital > Measure Risk > Set Risk Threshold > Analyze Risk Sources > Aggregate Risk > ***Determine Capital by Line of Business***

After the aggregate risk measure has been determined, the practitioner would then select the company's desired risk capital, based on whichever measure is deemed appropriate (V@R, EPD, probability of ruin, etc.) The remaining step is to determine how to split up that risk capital by line of business. The paper discusses four possible methods:

1. **Proportional Allocation Based on a Risk Measure:** This method is attractive because it is simple to apply and intuitively makes sense. It is easily explained via example:

   Suppose we have four sources of risk, which, based on some risk measure (V@R, TV@R, EPD) contribute on a standalone basis 1, 2, 3, and 4 dollars of risk capital, respectively. Then on a standalone basis, each source accounts for 10, 20, 30, and 40% of the total risk, again respectively. If the sources of risk are not all perfectly correlated, then the aggregate risk capital will be something less than 10, say 8. Using the proportional allocation, we would assign capital based on contribution to risk. The first source would get 10% of 8, or 0.8, and similarly, 1.6 to the second, 2.4 to the third, and 3.2 to the fourth.

   When applying this method, the method used to allocate capital can be the same as that used to determine the appropriate amount of aggregate risk capital, but does not necessarily have to be. For example, the company may choose to set aggregate risk capital based on the V@R(99) level, but to allocate risk capital based on relativities at the CTE(99.5) level. Choosing to allocate based on a different measure may (or may not) result in a substantially different allocation, depending on the shape of the distributions.

2. **Incremental Allocation:** The incremental allocation method discussed in this paper is similar to the *Merton-Perold marginal allocation method* presented in the Cummins paper, except that it can be applied to any risk capital measure, not just the insolvency put option (expected policyholder deficit).

   If we have three different risk sources, A, B, and C, to apply an incremental allocation procedure, we would determine the amount of risk capital required for risks A&B alone, risks B&C alone, and risks A&C alone, as well as the total amount of capital required for all three risks. Then:

   - $(\text{Risk Capital})_A = (\text{Risk Capital})_{A\&B\&C} - (\text{Risk Capital})_{B\&C\text{ alone}}$
   - $(\text{Risk Capital})_B = (\text{Risk Capital})_{A\&B\&C} - (\text{Risk Capital})_{A\&C\text{ alone}}$
   - $(\text{Risk Capital})_C = (\text{Risk Capital})_{A\&B\&C} - (\text{Risk Capital})_{A\&B\text{ alone}}$

   A key issue in this method is that this type of allocation is generally not additive – the sum of the capital allocated to each risk will not exactly equal the aggregate capital required for the business. There is no generally accepted practice for assigning the excess capital, nor is there a consensus as to whether the excess capital should be assigned at all.

3. **Myers-Read Marginal Allocation:** As noted in Cummins and again here, the incremental allocation method is not super reasonable because it implicitly assumes that firms will add or remove entire lines of business, rather than adjusting the volume of business. The other type of marginal allocation (Myers-Read) assumes the latter, which is more reasonable (although not perfect since it also assumes constant marginal costs, which is only appropriate in quota-share type arrangements).

The Myers-Read method looks to quantify the value of a firm's put option (the ability of the firm to fully or partially default on its obligations, in the event that losses exceed assets, and put the costs back to the policyholders). Myers-Read then looks to assign capital so that each risk source has the same marginal impact on the value of the put option.

A really nice thing about the Myers-Read method is that it perfectly allocates aggregate capital.

Some not-as-attractive things about this method are that:

- It is really intended to determine the frictional cost of each risk, not to assign risk capital.
- It is quantitatively intense.
- It is really not appropriate unless the risks exhibit homogeneity – that is, the shape of the loss distribution is invariant to changes in exposure.

The calculations for the MR method are shown in the Appendix – I won't repeat them, because the method is already discussed in the Cummins Capital Allocation paper.

4. **Co-Measures Approach:** The co-measures approach is similar to proportional allocation except that it does not use standalone risk measures, and assigns capital based on its contribution to the overall aggregate risk measure.

   For example, if the method of allocation is chosen to be V@R(99), and at the firm-wide V@R(99) level, sources A, B, and C cause losses of sizes 1, 2, and 3, for an aggregate loss of 6, the portion of the aggregate risk capital assigned to each of these sources would be 1/6, 2/6, and 3/6, respectively.

   As is the case with proportional allocation, the measure used for allocation need not be the same measure as that used for determining aggregate risk capital.

This concludes the discussion of how to determine capital to guide strategic decisions. To summarize, we have these steps to determine the capital needed:

Summary

1. Define capital (risk-adjusted measures include regulatory required, rating agency required, economic, and risk capital).
2. Measure risk (e.g., probability of ruin, V@R, CTE, EPD ratio).
3. Set risk threshold (bond default probability, management preference, arbitrary).
4. Analyze risk sources (market, credit, insurance, other).
5. Aggregate risk (determine dependency; aggregate distribution using closed form, approximation, or simulation).
6. Determine capital by line of business (proportional allocation, incremental allocation, Myers-Read, co-measures).

Now that we have determined capital, we can move onto the good part – applications. Some applications of risk-adjusted performance metrics are discussed in section 5 of the paper, and include the following:

1. **Assessing Capital Adequacy:** The aggregate risk profile and risk measures can help insurers identify whether the firm holds enough capital to meet policyholder/ regulatory obligations and whether management understands sources of risk and actively measures and manages its exposure to risk.

2. **Setting Risk Management Priorities:** Allocation methods can help firms to identify business units or activities which generate the greatest need for risk capital (and the greatest opportunity for risk management).

3. **Evaluating Alternative Risk Management Strategies:** RAROC can be used to test the impact of alternative strategies of risk reduction, such as when used to compare alternative reinsurance strategies.

4. **Risk-Adjusted Performance Measurement:** The risk-adjusted metrics can help to evaluate whether one line of business performed superior to another, after adjusting for risk, as done in the example below.

Example (From Table 22 of the paper): Suppose a company writes two lines of business. As measured at the end of the year, Line A had achieved a 92% loss ratio, and Line B an 86% loss ratio.

On first glance, Line B has a lower loss ratio, so performed better, but we've not considered the risk differences between the two businesses.

Consider other details regarding the policies as below:

	Line A	Line B
Premium	6,400,000	6,400,000
Expenses	320,000	320,000
Investment Return	304,000	304,000
Loss Ratio	92%	86%
Claim Costs	5,888,000	5,504,000
Economic Profit	**496,000**	**880,000**

Note: Economic profit is based on actual results, and equals

(Premium) – (Expenses) + (Investment Income) – (Losses)

Again, by just comparing economic profit, we can say that Line B outperformed Line A. But, if, based on whichever chosen capital allocation method, we had earlier assigned capital to each line as below (the selection of risk capital is developed throughout the paper), we could see that Line A actually outperformed B when considering the capital required to support it.

	Line A	Line B
Economic Profit	496,000	880,000
Allocated Capital	2,117,082	4,225,340
RAROC	**23.4%**	**20.8%**

Do note that the risk-adjusted measure does have the downfall of not necessarily producing consistent results when comparing, as the results may differ had we chosen a different method to allocate capital.

5. **Insurance Policy Pricing:** RAROC can be used to set a price such that expected RAROC is above a target rate (example to come). There are a few nuances to bear in mind when using RAROC in this manner:

   a. **Investment Income on Allocated Capital**: When setting a target rate of return, one should be certain as to whether that rate is defined to be inclusive or exclusive of investment income. In the example above, the return was measured in excess of what was already earned through investment income.

   b. **Multi-Period Capital Commitment**: When capital is exposed to risk for multiple periods, as is customary in most insurance applications, one should incorporate the effect of releasing capital to appropriately adjust the RAROC ratio or target rate. This can be done by assuming a pre-defined release pattern to determine the present value of the cost of risk capital (RC), or economic profit, which is:

$$\text{PV(Cost of RC)} = \sum_{\text{all } n} (\text{Year } n \text{ BOY RC}) \times (\text{Target Return on RC}) \times (\text{Discount Factor})$$

   In this formula, "BOY RC" is risk capital at the beginning of a period, and the cost of capital is based off the target return rate. Note that in the paper, Goldfarb effectively discounts to the "beginning of Year 0" – the first year's beginning risk capital is discounted a year (see Table 29).[9]

   You would then adjust the target return by the ratio of economic profit, determined using the formula above, to initial capital (capital at the beginning of year 1).

   An alternative to the economic profit method would be to assume a steady state, which would incorporate the reserve risk capital into calculations of initial required capital for each line of business, in addition to the underwriting risk already accounted for in the formula above.

   c. **Cost of Risk Capital**: When using RAROC, one must consider how the cost of risk capital is to be determined in the first place. Some issues here are:

   - **CAPM v. RAROC**: While a common textbook suggestion for determining RAROC is through the CAPM, this presents difficulty since they measure different forms of risk. The "risk" considered in CAPM stems from the exposure to systematic risk when added to a portfolio of diversified investments.

[9] This table is reproduced in the Drive in directory *Other Files* > ***Excel Supplements***.

On the other hand, the "risk" considered in RAROC is that between expected cash flow compared with cash flow in the tail of the probability distribution. Thus, using CAPM within the RAROC framework leads to some inconsistencies in capturing what is intended to be measured. (Furthermore, as we've seen in BKM, the "correctness" of the CAPM in the first place is questionable.)

- **RAROC is artificially leveraged**: As the capital defined in the denominator of RAROC considers neither the market value of invested capital nor actual capital that could be exposed to loss (committed capital), RAROC will necessarily appear higher than it is.

  The investment rate of return demanded by shareholders is dependent upon the market value of the firm's equity, which is generally in excess of book equity (due to franchise value). As a result, achieving the target rate of return solely on risk capital does not generally capture the full extent of return demanded by shareholders, who generally don't care about earning returns on risk capital, but rather on market value of equity.

If it is accepted that CAPM is in fact a reasonable measure of the risk considered in risk capital, then one compromise would be to adjust the CAPM return by the ratio of the firm's total capital to the firm's risk capital. Alternately, the RAROC calculation could allocate the firm's total capital rather than just the risk capital. One must also consider that a firm-based CAPM may not necessarily be appropriate for each individual line of business, as some are more exposed to systematic risk than are others.

Many other adjustments could be made to tweak RAROC into a more refined measure. For example, à la Feldblum, it can be adjusted to incorporate the frictional costs of holding capital (such as the costs of double taxation on investment income). This would perhaps represent an improved measure, though not complete, as it still does not address the risk that would need to be included even in the absence of taxes.

In another paper, Mango points out that allocation of capital is actually just an allocation of underwriting capacity, so each business unit must earn sufficient profits to pay for this capacity. Additionally, since each line of business has the right to call upon the entire capital of the firm to pay claims, the line must earn enough profits to compensate the firm for the value of this call option, so these costs could also be reflected in the cost of capital.

Example (From tables 25, 26, 27)**:** Given the following:

- Expected premium = 6,400,000
- Expected expense ratio = 5%
- Expected investment return = 5%
- Expected loss ratio = 91.6%
- Allocated capital (used a co-CTE allocation) = 4,225,340
- Target return on capital = 15%

The expected economic profit in this situation is determined as:

Premium	6,400,000	
Expenses	(320,000)	= 5%(6,400,000)
Investment Income	304,000	= 5%(6,400,000 – 320,000)
Claims	5,862,400	= 91.6%(6,400,000)
Expected Economic Profit	**521,600**	

The expected RAROC then would be

$$\frac{521{,}600}{4{,}225{,}340} = \mathbf{12.3}\%$$

Expected RAROC falls short of the target return of 15%, so we would need an additional risk margin in premium. We would like the economic profit to be:

$$4{,}225{,}340 \cdot 15\% = 633{,}801$$

$$633{,}801 = (6{,}400{,}000 + \pi - 320{,}000)(1.05) - 5{,}862{,}400$$

Solve for the additional risk margin, $\boldsymbol{\pi} = \mathbf{106{,}858}$.

This solution assumes that expenses don't vary with premium.

The general formula for RAROC, which we used above, is:

$$\textbf{RAROC} = \frac{\boldsymbol{(P + \pi - E)(1 + i) - L}}{\textbf{CAPITAL}}$$

In section 4 of the paper (and continued into section 5), Goldfarb presents a sample calculation. It's rather helpful so I'd recommend you review it – here I will just present a list of steps. (Some of the calculations were already shown in part here in the examples.)

1. States assumptions regarding invested assets, loss reserves, written premium by line of business, and expenses. Other risk sources are ignored for simplicity.
2. Simulates stand-alone risk distributions and then an empirical aggregate distribution.
3. Uses as selected measure for risk capital that which corresponds to the V@R(99) level.
4. Demonstrates several methods that could be used to allocate the total capital to each line of business.
5. Determines *ex post* RAROC based on actual economic profit as a ratio to allocated capital.
6. Demonstrates how RAROC can be used for insurance policy pricing, based on an assumption for expected loss ratio.

Practical Considerations

1. **Time Horizons:** As noted earlier in the paper, a discrepancy exists between the different time horizons used to measure market risk (usually a one-year horizon) versus insurance risk (usually measured to reflect risk of ultimate liability. Where this difference is substantial, it can be addressed by using dynamic financial analysis (DFA), although this substantially complicates calculations and adds a great degree of uncertainty to estimates.

   Another approach to address the issue would be to measure market risk and *change in value* of insurance liabilities on the same one-year horizon. Again, this is not a perfect solution as information about re-evaluation of liabilities may not be available, and even when available, the change in value may be miniscule, even when there is substantial risk over the longer horizon, as is often the case with high-layer excess general liability.

   Still another, and perhaps the most common, approach is to ignore the discrepancies, which although not as mathematically sound, is quite a bit simpler for modelling purposes.

2. **Alternative Return Measures:**
   - The paper focused on the use of a RAROC return measure to reflect economic profit, though alternative measures could be used. Accounting measures provide a reasonable alternative if one would want to use a measure with which senior management is familiar.
   - Taxes could be incorporated to allow for a more realistic, albeit more complex, measure.
   - Stranded capital (the excess of the firm's total capital versus total allocated risk capital) could be reflected with a reduction of the rate of return. This is conceptually similar to the idea discussed earlier of allocating the firm's total capital rather than just the risk capital.
   - Investment income can be reflected over multiple periods by using present values.

3. **Risk-based Allocation:** The methods of allocation relied upon tail measures of risk, and therefore drive the most capital to those lines that are most heavily skewed. This is in line with regulatory measures, but not necessarily shareholder demands, which are more focused on changes in credit rating or financial strength.

4. **Diversification Adjustments:** It was mentioned earlier that correlation and dependency play substantial roles in determining the firm's aggregate risk profile, and unfortunately also tend to be quite sensitive to estimation errors. In light of this, while RAROC is still an informative measure, it should not necessarily be used as the sole metric to drive managerial decisions.

Bodoff (Capital Allocation by Percentile Layer)

When firms look to determine an appropriate amount of capital and target return on that capital, they must consider not only the characteristics on their particular lines of business, but also external forces, among them regulators, rating agencies, and investors. Given that the firm must hold capital, they can consider this capital in the same way as any other "overhead" cost, which similarly needs to be allocated appropriately. The method a firm chooses to allocate capital can affect profitability, target pricing margins, and volume of business.

In his simple scenario, Bodoff uses a firm choosing to hold capital at the frequently-used V@R(99) level. The underlying issue with this methodology is that the firm holds capital sufficient for a catastrophic loss, but neglects to consider the potential need for capital to support any other losses.

Similarly, the slightly more refined TV@R(99) considers all losses above the selected level (as a straight average), but disregards everything under that level. The main issue with this is that, for example, if the loss corresponding to the selected V@R level is $100, a loss of $99 would not be considered relevant in capital allocation.

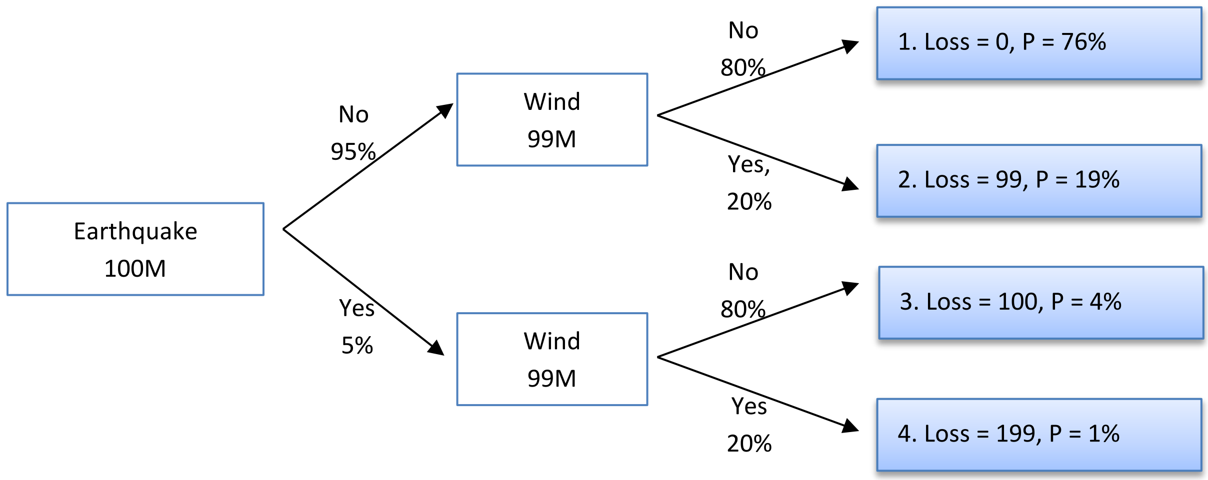
The purpose of the paper to present an alternate method for allocating capital that would reflect that all losses contribute to capital depletion.

Bodoff's Thought Experiment #1

Bodoff explains his process using some simplified examples. He posits a world with two possible perils:

Peril	Probability	Loss
Earthquake	5%	100 M
Wind	20%	99 M

This gives us the following universe of possibilities:



We might choose any of several common methods for allocation, assuming we want to hold enough capital to fund the V@R(99) level (which is 100M):

	Portion Allocated to	
Allocation Method	Wind	Earthquake
CoVar Allocate to the only peril that pierces the 100M level	**0%**	**100%**
Alternative CoVaR[10] Allocate to events (3) and (4), since both pierce 100M. Allocate to each event based on likelihood, and allocate to each loss within an event based on severity.	$\left(\frac{4}{4+1}\right)\left(\frac{0}{100}\right) + \left(\frac{1}{4+1}\right)\left(\frac{99}{199}\right)$ **= 9.95%**	$\left(\frac{4}{4+1}\right)\left(\frac{100}{100}\right) + \left(\frac{1}{4+1}\right)\left(\frac{100}{199}\right)$ **= 90.05%**
CoTVaR Allocate by probability and dollars, instead of by probability alone. Allocate to each loss by severity, as before. Then re-scale to desired capital level. Allocate to events (3) and (4) as follows: Event 3: $\mathbf{80}\% \times \mathbf{100M}$ Event 4: $\mathbf{20}\% \times \mathbf{199M}$	$80\% \cdot 100\left(\frac{0}{100}\right) + 20\% \cdot 199\left(\frac{99}{199}\right) = 19.8$ $\frac{19.8}{100+19.8} = \mathbf{16.5}\%$	$80\% \cdot 100\left(\frac{100}{100}\right) + 20\% \cdot 199\left(\frac{100}{199}\right) = 100$ $\frac{100}{100+19.8} = \mathbf{83.5}\%$

Each of these three allocations demonstrates the major drawback of any traditional method of allocation – neither allocated particularly large portions to wind (in fact, no capital is allocated to the "wind only" event), although that peril is much more likely than the earthquake peril and would cause nearly as catastrophic a loss.

Defining a Percentile Layer of Capital

Bodoff defines a **percentile layer of capital** and a **layer of capital** in an intuitive way – it simply references the amount of capital required at a given requirement level. For example, if 100 losses are simulated, and the 77th smallest loss is 47M and the 78th smallest loss is 59M, then the percentile layer of capital (77%, 78%) would be the 12M layer of capital between 47M and 59M.

> **Bodoff's Capital Allocation Methodology**
>
> For each layer of capital, take the amount of capital ("width" of the layer), and allocate to only events piercing the layer, by the conditional probability of penetrating the layer.

[10] This is the co-measure V@R approach discussed in Goldfarb's paper.

A desired quality of a capital allocation methodology is that it, for each layer, allocates capital only to those losses piercing the layer. Continuing the above, losses 77 through 100 should each be allocated some of the 12M loss in the (77%,78%) layer, but none of the smallest 76 losses should receive any capital allocation from that layer.

Therefore, Bodoff's methodology simply allocates the capital in a given layer, to each event piercing the layer, weighted by the probability that the loss penetrates the layer. This methodology necessarily results in an allocation that exactly equals the total amount of capital, so removes the need to proportionately scale allocated losses later.

We go back to the original Thought Experiment to demonstrate the methodology in action.

Application of Capital Allocation by Layer to Thought Experiment #1

Recall that we desire to allocate 100M of capital. We are interested in each layer from 0 to 100M that separates the loss events. Our bottommost layer then is from 0 to 99M, followed by the remaining layer from 99 to 100M.

We allocate to each event based on the conditional probability of entering the layer.

		Event (Loss Size, Probability)			
Layer	**Width**	**Event 1** **0, 76%**	**Event 2** **99M, 19%**	**Event 3** **100M, 4%**	**Event 4** **199M, 1%**
0 to 99 M	99 M	Not in layer	$^{19}/_{19+4+1} = {}^{19}/_{24}$	$^{4}/_{24}$	$^{1}/_{24}$
99 to 100 M	1 M	Not in layer	Not in layer	$^{4}/_{4+1} = {}^{4}/_{5}$	$^{1}/_{4+1} = {}^{1}/_{5}$

Note that the total width of layers adds to 100M, and the probability within each layer sums to 1.

So, the total capital allocated by event is:

- Event 2: $19/24 \times 99 = 78.4\text{M}$
- Event 3: $4/24 \times 99 + 0.8 \times 1 = 17.3\text{M}$
- Event 4: $1/24 \times 99 + 0.2 \times 1 = 4.3\text{M}$

Again note that the capital sums exactly to 100M.

We could allocate event 4 down further based on loss size. Recall Event 4's 199M came from 99M from the wind event and 100M from earthquake.

So our final allocation to wind becomes: $78.4 + \frac{99}{199} \times 4.3 = \mathbf{80.5M}$

And our final allocation to earthquake is: $17.3 + \frac{100}{199} \times 4.3 = \mathbf{19.5M}$

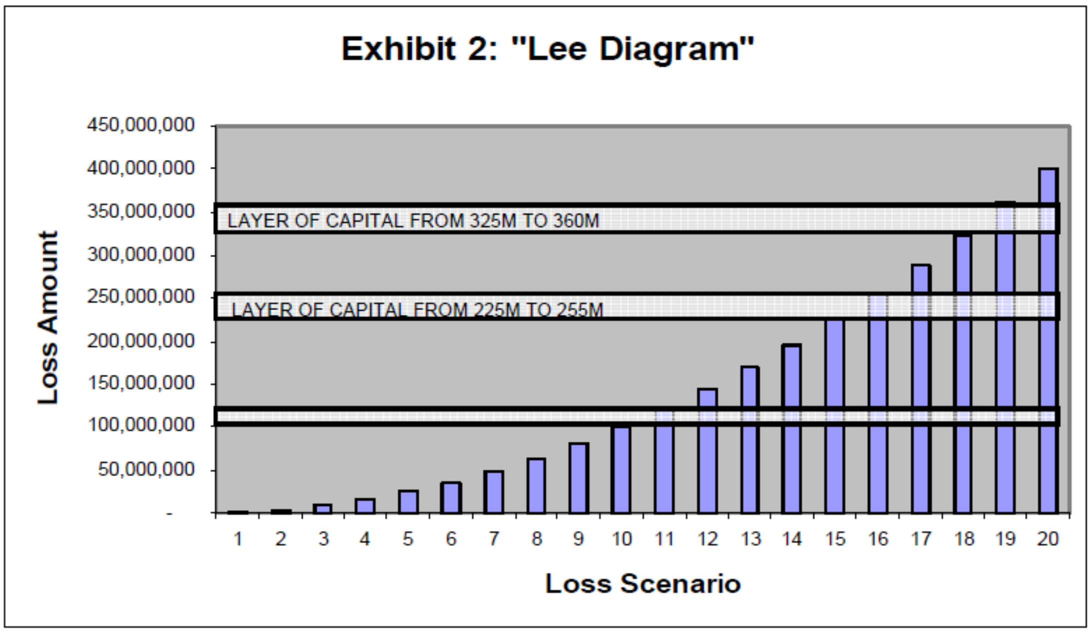
The big takeaway here is that the capital allocation by layer methodology results in a substantially greater allocation to wind. This is desirable since wind has a greater likelihood of occurrence (and is similarly catastrophic to the earthquake event).

In the example, Bodoff chose to use V@R, but he notes that the method could also work for a TV@R approach, except that in the latter case, one would have to also allocate the additional layer of capital represented by [TV@R – V@R] to those losses that exceed the TV@R threshold, allocated to events in proportion to each event's average amount of loss excess of the TV@R threshold.

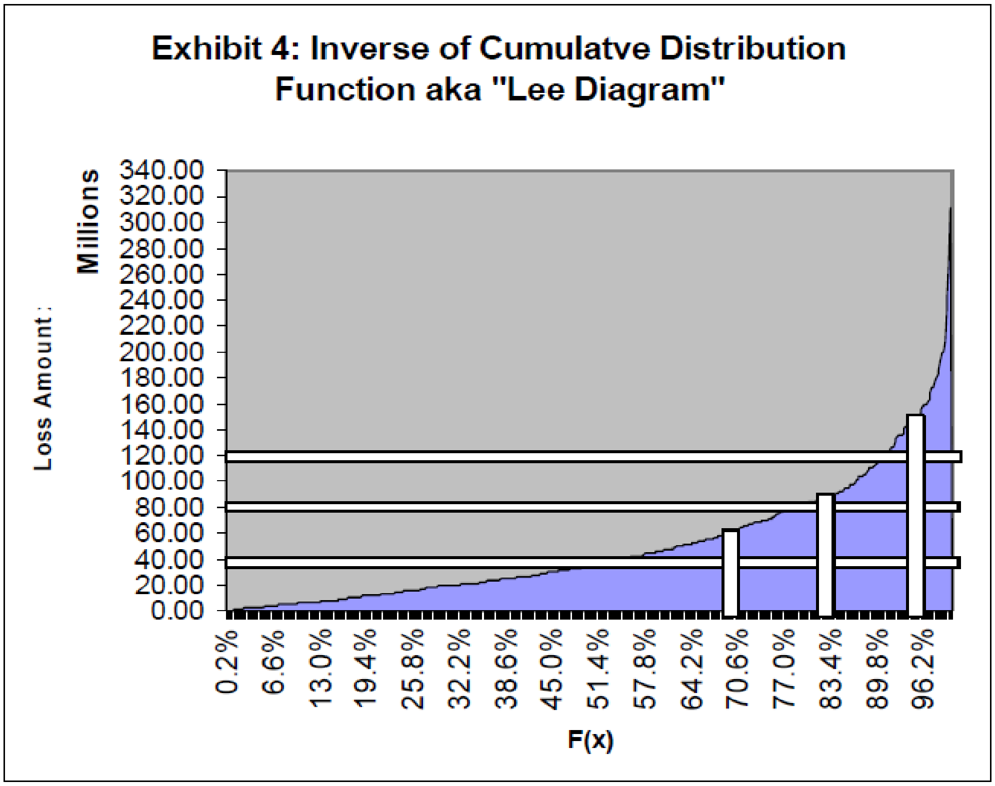
Graphical Depiction of Capital Allocation by Percentile Layer

In Bodoff's Exhibit 2, he demonstrates how a Lee Diagram can be used to visualize certain properties of the capital allocation by layer method. We are shown a discrete distribution of 20 events, and if we choose to allocate at the V@R(95) level, we look to distribute 360M to all scenarios generating any loss in the layers leading up to 360 M. The horizontal bars demonstrate that a loss event tends to receive a larger percentage allocation in the upper layers for two reasons:

1. There are fewer losses piercing the layer, so a proportionately larger amount goes to each loss that does pierce.
2. The larger layers tend to have greater widths because generally the loss difference from a large loss to a larger loss tends to grow greater toward the end of the distribution (depending on the distribution, but fairly typical for insurance applications).



Bodoff extends his Lee Diagram to the continuous case to demonstrate that the method can be thought of in two different ways:



Horizontal Procedure: For each layer of capital, allocate to all events that penetrate the layer. This is shown by the horizontal bars in Bodoff's Exhibit 4, reproduced above. We can express this as an integral, starting horizontally (with respect to x) and then allocating to each layer (integrating with respect to y). We get:

$$\int_{y=0}^{y=\text{VaR}(99\%)} \int_{x=y}^{x=\infty} \frac{f(x)}{1-F(y)} dxdy$$

Vertical Procedure: For each loss, allocate capital for all layers it penetrates. The integral is equivalent except for change of integration order:

$$\int_{x=x(0\%)}^{x=\infty} \int_{y=0}^{y=\min(x,\text{VaR}(99))} \frac{f(x)}{1-F(y)} dydx$$

Allocated Capital

This formulation leads to some important results. We can state that, for each loss, we are allocating capital, and define the allocated capital, AC, to be:

$$AC(x) = \int_{y=0}^{y=x} \frac{f(x)}{1-F(y)} dy = f(x) \int_{y=0}^{y=x} \frac{dy}{1-F(y)}$$

Note that for simplification purposes, the x used here represents the full value of x, capped at the chosen capital level (e.g., V@R(99)). Viewing the allocated capital to each loss in this way, we can see that the loss's allocated capital depends on three things:

1. The probability of the event occurring, $f(x)$
2. The severity of the loss event (the extent to which loss penetrates layer of capital), the upper bound of integration
3. The loss event's inability to share the burden with other loss events $\int \frac{1}{1-F(y)} dy$

Taking the derivative of allocated capital with respect to x can also lead us to some other conclusions regarding the relationship of loss and allocated capital, as described by formula (6.10) of the text:

$$\frac{d}{dx}[AC(x)] = \frac{f(x)}{1-F(x)} + f'(x) \int_{y=0}^{y=x} \frac{dy}{1-F(y)}$$

1. As loss amount increases, allocated capital increases because the loss receives allocation from an additional layer of capital $= \frac{f(x)}{1-F(x)}$
2. As loss amount increases, allocated capital decreases because the loss amount is less likely to occur, so it receives a lower allocation on lower layers of capital $= f'(x) \int_{y=0}^{y=x} \frac{dy}{1-F(y)}$
   a. Since $f(x)$ describes the probability of a loss size of x, it is usually a decreasing function of x, and so $f'(x)$ is typically negative.
   b. For discrete events, $f(x)$ is a constant $\frac{1}{n}$ with derivative $f'(x) = 0$, zeroing out the second term.

We can take the allocated capital a step further. Bodoff describes the **disutility** (pain) of an event *x* to be the product of allocated capital and required rate of return on capital, *r*.

$$\text{cost of capital} = r \cdot f(x) \int_{y=0}^{y=x} \frac{1}{1-F(y)} dy$$

If we are given that the loss has occurred, the probability of occurrence drops out, so we have:

$$\text{cost of capital, given loss x} = r \int_{y=0}^{y=x} \frac{1}{1-F(y)} dy$$

And so the total cost of the event is the loss itself plus the cost of capital:

$$\text{Total cost of event x} = x + r\int_{y=0}^{y=x}\frac{1}{1-F(y)}dy$$

Applying to Premium

We can use these results to determine an appropriate level of premium. Bodoff ignores expense for purposes of this presentation. So we have:

$$\begin{aligned}\text{Premium} &= \text{Expected Loss} + \text{Cost of Capital}\\ &= \text{E[L]} + r(\text{allocated capital} - \text{contributed capital})\end{aligned}$$

Since contributed capital = net premium, we have: $P = E[L] + r(AC - P)$. After some manipulation, this is equivalent to:

$$\boldsymbol{P = E[L] + \frac{r}{1+r}(AC - E[L])}$$

Using the previous formulation of allocated capital, we then have:

$$P(x) = xf(x) + \frac{r}{1+r}\left[f(x)\int_{y=0}^{y=x}\frac{dy}{1-F(y)} - xf(x)\right]$$

$xf(x)$: Expected Loss

r: Return on capital

$f(x)\int_{y=0}^{y=x}\frac{dy}{1-F(y)}$: Allocated capital

The article rearranges terms and reformulates to arrive at a few conclusions:

1) Required premium associated with loss event x is expected value multiplied by what can either be considered as an adjustment to the loss amount, or an adjustment to the probability of occurrence.
2) Risk load increases with respect to loss amount, at an increasing rate.
3) Even for very small values of loss event x, the risk load is positive (even for losses less than the mean. This is because while the expected value of a loss may be less than the mean, if we are given that the loss does occur, the impact could be greater than the mean, and therefore should receive some allocation of capital.)

Conclusions

Capital allocation by layer has several advantages:

1) It emerges organically from a new form of meaning of holding Value-at-Risk capital.
2) It allocates capital to the entire range of loss events rather than just the tail.
3) It allocates more capital to events that are more likely or are more severe.
4) It produces allocation weights that are additive and explicitly allocate the entire amount of the firm's capital.
5) It provides a framework for allocating capital by layer and tranche.

Addendum 1: Reconciliation of Bodoff's Premium Formulation with that of Panning

Recall that Panning expressed premium as:

$$P = \frac{S(k-y)+L}{1+y} + E$$

This is equivalent to:

$$\boldsymbol{P - L - E + (S + P - E)y = kS}$$

Bodoff uses:

$$P = E[L] + \frac{r}{1+r}(AC - E[L])$$

This is equivalent to:

$$\boldsymbol{P = E[L] + r(AC - P)}$$

The below will show how to reconcile the second equation with the last.

1. Since Bodoff's premium is a **net premium** (net of expenses), adjust Panning to remove expenses, and now Panning's P (denoted $\tilde{P}$ to disambiguate) represents the same quantity as Bodoff's:

$$\tilde{P} - L + \left(S + \tilde{P}\right)y = kS$$

$$\tilde{P} = \frac{S(k-y)+L}{1+y} = \frac{S(k-y)}{1+y} + \frac{L}{1+y}$$

2. Bodoff doesn't explicitly reflect timing of loss payments, where Panning assumes losses are paid at the end of the year, so define $E[L] = \frac{L}{1+y}$, so that discounting is explicitly reflected. Then:

$$\tilde{P} = \frac{S(k-y)}{1+y} + E[L]$$

3. Panning assumes that surplus will be invested at the "target rate," which I am using to align with Bodoff's verbiage of target return = r. In Panning, the target rate is expressed as an excess over the risk-free rate, and discounted to reflect the time value of money. Bodoff does not do this, so denote $r = \frac{k-y}{1+y}$, giving:

$$\tilde{P} - E[L] = rS$$
$$\tilde{P} = E[L] + rS$$

4. Denoting S, surplus (excess contributions) as allocated capital minus contributed capital) = $AC - \tilde{P}$ we get to Bodoff's result:

$$\tilde{P} = E[L] + r\left(AC - \tilde{P}\right)$$

In summary, the difference between Bodoff and Panning's formulations are:

- Bodoff explicitly excludes expenses by defining premium as net of expenses
- Bodoff does not explicitly account for the time value of money

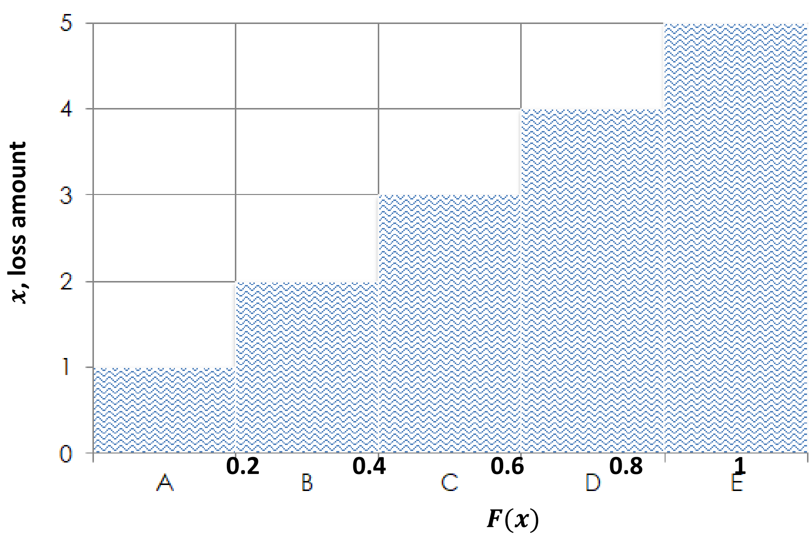
Addendum 2 – Additional Examples

These examples are intended to help in interpreting the integration formulas in this paper. I'm of the impression that evaluating the integrals are too much of an algebraic pain to be tested in the continuous cases, but who knows. In any case, you should still try to get a good feel for what the integrals are doing.

Discrete Example

Suppose we have five possible events, A – E, with respective size 1 – 5, and all equally likely.

Graph looks like this:



Note that,

For y = level of capital:

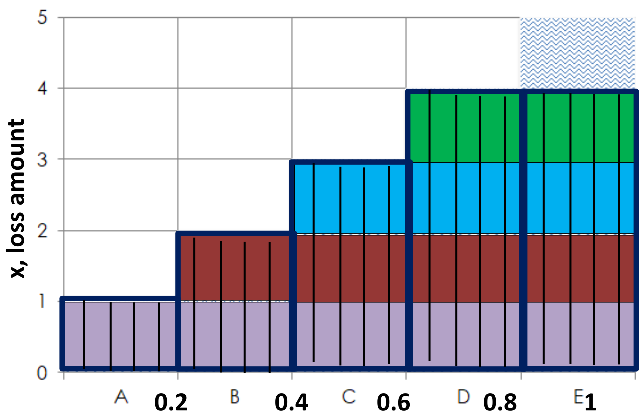
$F(y)$ defines the percentile associated with a given level of capital.

$$F(y) = \frac{y}{5}:$$
$$F(0) = 0$$
$$F(1) = 0.2$$
$$F(2) = 0.4$$
$$F(3) = 0.6$$
$$F(4) = 0.8$$
$$F(5) = 1$$

For example, $F(3) = 0.6$ means that to satisfy a V@R of 60%, we'd need capital of 3.

Capital Allocation by Layers Using Vertical then Horizontal Approach

If we want to allocate 4 in loss capital, the vertical then horizontal approach would have us determine the capital allocated to each event, and then add all the events together. For example, the capital allocated to Event C would be the sum of the purple, mauve, and blue boxes above Event C.



We allocate capital to <u>each event</u> based on the following:

$$AC(x) = \int_{y=0}^{y=x} \frac{f(x)}{1-F(y)} dy$$

Let's allocate up to capital of 4. Keeping in mind that each loss size has probability 0.2:

$$AC(x) = 0.2 \int_0^4 \frac{1}{1-F(y)} dy$$

We split up the integral by layer of capital; in this case, each layer has height 1.

$$AC(x) = 0.2\left[\int_0^1 \frac{1}{1-F(y)} dy + \int_1^2 \frac{1}{1-F(y)} dy + \int_2^3 \frac{1}{1-F(y)} dy + \int_3^4 \frac{1}{1-F(y)} dy\right]$$

We use $F(y)$ based on above.

$$AC(x) = 0.2\left[\int_0^1 \frac{1}{1} dy + \int_1^2 \frac{1}{0.8} dy + \int_2^3 \frac{1}{0.6} dy + \int_3^4 \frac{1}{0.4} dy\right]$$

$$AC(x_E) = AC(x_D) = 0.2\left[1 + \frac{5}{4} + \frac{5}{3} + \frac{5}{2}\right] = 1.283$$

$$AC(x_C) = 0.2\left[1 + \frac{5}{4} + \frac{5}{3}\right] = 0.783$$

$$AC(x_B) = 0.2\left[1 + \frac{5}{4}\right] = 0.45$$

$$AC(x_A) = 0.2[1] = 0.2$$

You should note that the bracketed values are **vertically summing** the boxes for a given event. Taking the sum of the events (**horizontally summing**) would give the total capital allocated, and is akin to the integral defined as:

$$\int_{x=0}^{x=\infty} AC(x)\, dx = \int_{x=0}^{x=\infty} \int_{y=0}^{y=x} \frac{f(x)}{1-F(y)} dy\, dx$$

The sum in this case of the total allocated capital is 1.283 + 1.283 + 0.783 + 0.45 + 0.2 = 4, as expected.

Horizontal Then Vertical Approach

Using $AL(y)$ to represent the losses allocated to capital layer y:

$$AL(y) = \int_{x=y}^{x=\infty} \frac{f(x)}{1 - F(y)} dx$$

$$= \frac{5}{5 - y} \int_{x=y}^{x=5} 0.2dx = 1$$

The double integral would be evaluated as:

$$\int_{y=0}^{y=4} 1\, dy = 4$$

In this method, you will always get $AL(y) = 1$, since you are determining the sum of the weights to each loss.

Continuous Example – Uniform Losses – Using Vertical then Horizontal Approach

Suppose the set of losses are uniform on [0, 10].

Note that this means that $f(x) = \frac{1}{10}, F(x) = \frac{x}{10} \rightarrow 1 - F(x) = \frac{10-x}{10}$.

We will be using $\frac{\mathbf{1}}{\mathbf{1-F(y)}} = \frac{\mathbf{10}}{\mathbf{10-x}}$ in our formula.

To determine how much capital is allocated to each loss size, we would have:

$$AC(x) = \int_{y=0}^{y=x} \frac{f(x)}{1-F(y)} dy = \frac{1}{10} \int_{y=0}^{y=x} \frac{10}{10-y} dy = \int_{y=0}^{y=x} \frac{1}{10-y} dy$$
$$= \ln(10-x)|_x^0$$
$$= \ln(10) - \ln(10-x)$$
$$= \ln\left(\frac{10}{10-x}\right)$$

Then, the total amount of capital allocated across all events is determined by integrating again, with respect to losses.

$$\int_{x=0}^{x=\infty} \ln\left(\frac{10}{10-x}\right) dx$$

For example, if we choose to allocate 9 in capital, then break up into losses above and below 9.

- To those losses up to size 9, we would allocate a total of:

  $$\int_{x=0}^{x=9} \ln\left(\frac{10}{10-x}\right) dx \rightarrow \cdots \rightarrow = 9 - \ln(10)$$

  I left out the calculus showing the integration of parts here because (1) it's really long and (2) for that reason it's not a reasonable exam question. It's on the following page if you are curious.

- For those losses above 9, the allocated capital to each would be capped:

  $$AC(x) = \ln(10-x)|_x^9$$
  $$= \ln(10) - \ln(10-9)$$
  $$= \ln(10)$$

  The total amount of capital allocated would be:

  $$\int_{x=9}^{x=10} \ln(10)\, dx = \ln(10)$$

- The sum of the allocated capital to all possible events (from loss size 0 to 10) is 9, as desired.

Showing the work from the ellipsis here:

$$\int_{x=0}^{x=9} \ln\left(\frac{10}{10-x}\right) dx \to \cdots \to = 9 - \ln(10)$$

1. Define $t(x)$ as the indefinite integral.

$$t(x) = \int \ln\left(\frac{10}{10-x}\right) dx$$

2. Use integration by parts: $u = \ln\left(\frac{10}{10-x}\right)$; $dv = 1dx$

   Then $du = \frac{10 \div (10-x)^2}{10 \div (10-x)} dx = \frac{1}{10-x} dx$; $v = x$

   So we get:

$$x \ln\left(\frac{10}{10-x}\right) - \int \frac{x}{10-x} dx$$

3. Use u-substitution on the integral, let $u = 10 - x$; du = -dx

$$-\int \frac{x}{10-x} dx = -\int -\frac{10-u}{u} du = \int \left(\frac{10}{u} - 1\right) du = 10 \ln|u| - u + C$$
$$\to 10 \ln|10-x| - (10-x) + C$$

4. Putting together and ignoring the constant, we have:

$$T(x) = x \ln\left(\frac{10}{10-x}\right) + 10 \ln|10-x| - (10-x)$$

5. Evaluating at the boundaries:

$$T(9) = 9 \ln(10) + 10 \ln(1) - 1 = 9 \ln(10) - 1$$
$$T(0) = 0 + 10 \ln(10) - 10 = 10 \ln(10) - 10$$

6. Subtracting $T(9) - T(0) = 9 - \ln(10)$ ■

Cummins (Allocation of Capital in the Insurance Industry)

This article, like a few others in the syllabus, presents a viewpoint of capital allocation. Why do we care so much about capital allocation? Because it can affect pricing and project selection. Also, insurance solvency is particularly important to policyholders because they cannot diversify away this risk – most people do not take out multiple auto policies, for example.

Elsewhere in the syllabus you will see or have seen that the allocation of capital is largely artificial – an insurer cannot declare bankruptcy on just one particular line, and all of the capital is available to the insurer to pay claims arising from any specific policy. It is important to note this – while capital allocation has several business uses, capital is never *really* allocated in real life.

One use of capital allocation is related to value maximization. While Cummins notes that this objective is frequently disregarded in favor of GAAP equity, the firm should also look to maximize market value, which is where capital allocation should be considered. Capital allocation helps the firm to measure performance by each line of business to determine how much, if any, value is added to the firm.

Cummins describes *the capital allocation problem* as one that we will see in other places in the syllabus – the sum of the capital allocated to all of the firm's businesses is subadditive – it will generally be less than the firm's total capital. Regardless of how that issue is settled, firms can look to maximize firm value by considering the **risk-adjusted return on capital (RAROC),** which is defined exactly as it sounds, as a ratio of the net income from a given line of business with respect to the (risk) capital it has been assigned.

$$\text{Risk} - \text{Adjusted Return on Capital } (\text{RAROC}_\text{i}) = \frac{(\text{Net income})_i}{C_i}$$

Net income should be after taxes and interest expense. A line of business' return is adequate if the RAROC exceeds the cost of capital. If RAROC is less than the cost of capital, it is eroding business value, and the firm should consider methods to address this, such as tightening underwriting standards, re-pricing, or withdrawing from the business entirely.

An equivalent method of determining whether a line of business adds value is **economic value added (EVA)**. EVA measures the excess of income over the required income, as determined by the cost of capital rate r_i, also called the hurdle rate.

$$\text{Economic Value Added } (\text{EVA}_\text{i}) = \text{Net income}_i - r_i C_i$$

A positive EVA indicates a value-adding line of business. An alternate formulation for EVA is **economic value added on capital (EVAOC)**, which is simply $EVA_i \div C_i$.

Now that we've seen how to use capital to measure profitability, it is helpful to know how to determine risk capital in the first place. One approach is the "pure play" technique, which estimates the cost of capital by comparing to monoline businesses. This is theoretically sound but complicated in practice because it is difficult to find firms that write only one line of business, and even more difficult to match levels of underwriting risk characteristics. Other potential metrics, like Value-at-Risk (V@R), RAROC, and EVA may be difficult to implement due to lack of data.

The paper discusses a variety of possible capital allocation methods – RBC, CAPM, V@R, and Marginal Allocation.

Capital Allocation Technique – Regulatory Risk-based Capital

Risk-based capital (RBC) is used to define the minimum capital it must hold to avoid regulatory intervention. The RBC system creates levels of intervention based on the ratio of a firm's total adjusted capital to its RBC. Firms can use RBC to allocate capital, though Cummins notes several shortcomings:

- The model is purely empirical, with no theory backing its creation.
- The formula disregards correlations between lines of business.
- Regulatory charges are of questionable accuracy – some of the charges are based on worst-case scenarios rather than statistics[11].
- Charges are based on book values, not market values.
- Charges ignore significant sources of risk (such as duration, convexity, and derivative-related).
- Charges are based on the average firm, so even if they were accurate, they may not be appropriate for a particular insurer.

The author provides some background into how RBC is calculated, and its components, in a painful reminder of Exam 6.

There are six components of the property & casualty RBC model:

- R_0: risk-based capital for holdings of stocks of the firm's subsidiaries
- $R_{1:}$ investment risk (stocks, bonds)
- R_2: loss reserve risk
- R_3: written premium risk
- R_4: credit risk (default risk)
- R_5: off-balance sheet risk

The RBC formula is composed of the sum of the subsidiary RBC and the covariance adjustment, the latter of which disregards any correlation between risks 1 through 5.

$$RBC = R_0 + \sqrt{R_1^2 + R_2^2 + R_3^2 + R_4^2 + R_5^2}$$

Despite the shortcomings of the RBC method, it's still important to consider, since it does incorporate some important risks, and helps the firm to consider applicable regulatory restrictions.

[11] RBC charges are no longer based on worst-case scenarios, but on the 87.5 percentile of scenarios.

Capital Allocation Technique – Capital Asset Pricing Model

We can use our old friend the CAPM to determine capital. This is nice in that the CAPM is widely known, and managers might prefer it. Using CAPM, the expected return on equity is given by:

$$r_E = r_f + \beta_E(E[r_M] - r_f)$$

In this formulation,

- r_E = cost of equity capital
- r_f = risk-free rate
- $E[r_M]$ = expected market return
- β_E = firm's beta, given by $\text{Cov}(r_E, r_M) \div \sigma_M^2$

Using the CAPM to allocate to each line of business, the required rate of underwriting return would be given by:

$$r_i = -k_i r_f + \beta_i(r_M - r_f)$$ [12]

In this formulation,

- k_i represents the liability leverage ratio for line i, (i.e., liability ÷ equity);
- $-k_i r_f$ represents the interest paid for the use of policyholder funds based on the systematic risk of the line, $\beta_i(r_M - r_f)$.

Example: Given the following information we can use the CAPM to determine the required rate of underwriting return for the line of business:

- Risk-free rate of return = 3%
- Market risk premium = 7%
- Line beta = 0.8
- Line's liability leverage ratio is 40%.

Then, $r_i = -k_i r_f + \beta_i(r_M - r_f) = -0.4(3\%) + 0.8(7\%) = \mathbf{4.4}\%$.

The CAPM result implies that we need not allocate capital by line using the CAPM but charge each line for at least the CAPM cost of capital, as determined by the line's beta and leverage ratio. We could also use cost of capital based on some other asset pricing model, like the arbitrage pricing theory.

[12] The paper uses r_m instead of $E[r_m]$ to refer to expected market return; that is a little imprecise.

Despite the comfort from its familiarity, the CAPM method comes with some major flaws:

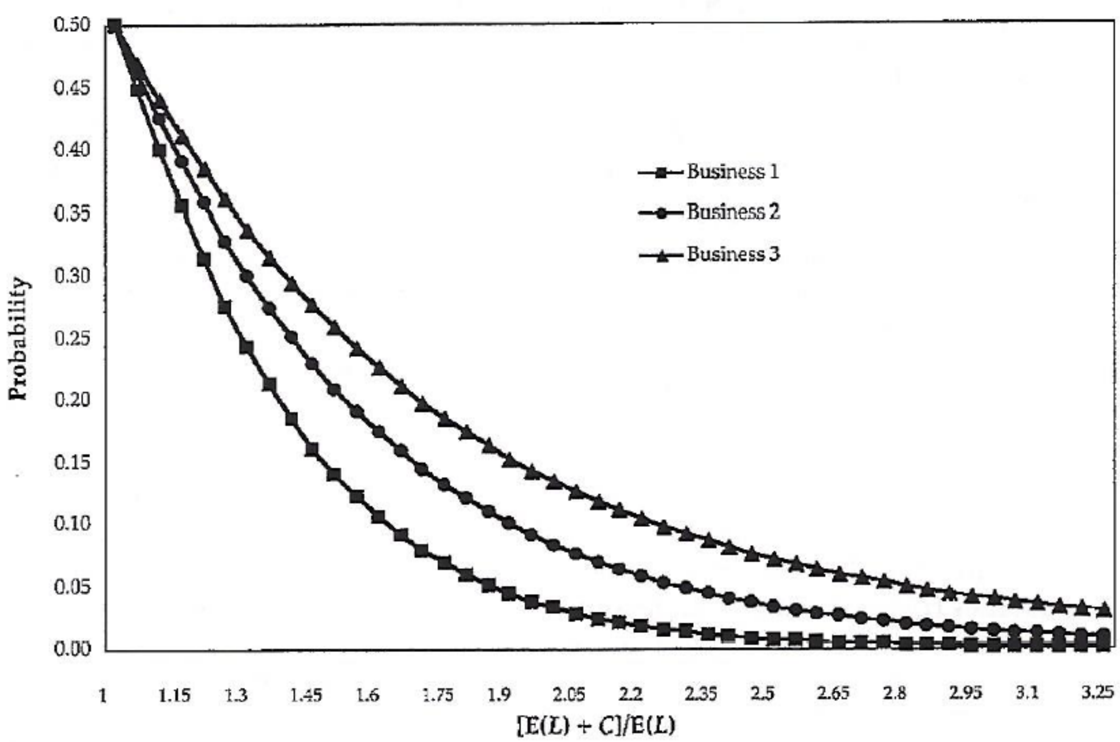
- It only considers systematic underwriting risk, and not the extreme events that are the major components for risk for insurers.
- It is difficult to estimate betas for entire firms, let alone betas for a given line of business.
- The underlying theory of the CAPM is not necessarily sound – we saw in the BKM text that CAPM ignores other important factors (such as those considered in the Fama French 3-Factor Model).

Capital Allocation Technique – Value-at-Risk

Value-at-Risk (V@R, VaR) is a common measure in insurance applications, and reflects the maximum amount the firm could lose over some time period, to a remote probability. V@R(99) would reflect the loss corresponding to the worst 1% of events. V@R techniques can be difficult to implement because they require very frequent data and robust data processing and information systems. V@R can be used in capital allocation through the use of exceedance probabilities, $\varepsilon = \Pr[\text{Loss}_i > \text{E}(\text{Loss}_i) + C_i]$.

In the description of Figure 1 from the paper (reproduced below), the author writes that loss is plotted against probability of loss, which is the case in a traditional loss exceedance curve. The axes are labelled for the alternate interpretation in the paper, which is the ratio of expected loss plus capital to expected loss. Business 3's (2.8, 5%) point implies it would need to commit $1.8 in capital for every dollar of expected loss to get down to 5% exceedance probability.

FIGURE 1
VaR Exceedence Probability Curves



While value-at-risk is a common go-to for capital allocations, it too comes with some drawbacks:

- The firm may not have enough capital to attain the desired exceedance probability level.
- V@R does not always consider diversification effect.
- V@R disregards the severity of losses in the tail.

Capital Allocation Technique – Insolvency Put Option/ EPD

The insolvency put option is what Butsic refers to as the *expected policyholder deficit*. Recall that when the EPD formulas for lognormal and normal distributions were provided, they were based on values derived from options pricing theories. The value of the policyholder's claim on the firm's assets is the equivalent of the present value of liabilities (at the risk-free rate), minus the put option. Symbolically:

$$\text{Value of Policyholders' Claim} = Le^{-rt} - \mathrm{P}(A, L, r, \tau, \sigma),$$

where $\mathrm{P}(A, L, r, \tau, \sigma)$ is the value of the put option on assets A with strike price L, interest rate r, time to maturity τ, and risk parameter σ to reflect the volatility of assets and liabilities, as well as the correlation between them. À la Butsic, $\mathrm{P}(A, L, r, \tau, \sigma)$ is the expected policyholder deficit.

As noted in Butsic, the key advantage of this method over value-at-risk is that it considers severity of extreme outcomes. We can allocate capital such that the EPD ratios for each line are equalized at a specified target level. This is an improvement, but it still fails to consider diversification across business lines.

Capital Allocation Technique – Marginal Capital Allocation

Like the EPD, marginal allocation is based on the option pricing model of the firm. This model views the value of the policyholders' claim on the firm as equal to the present value of losses, less the value of the insolvency put option. The expected loss to policyholders can be viewed as an insolvency put option because the policyholders have a claim on liabilities but cannot collect more than the value of the firm assets. A key improvement reflected in the marginal capital allocation methods is the recognition of the benefit of diversification.

Two methods of marginal capital allocation are **Merton-Perold (MP)** and **Myers-Read (MR)**. The MP method considers the marginal portion as calculated when removing entire lines of business, while MR considers the marginal portion as something akin to removing very small components of the lines of business. MR is more mathematically beautiful because it allocates 100% of capital, while MP produces a subadditive result.

Marginal Capital Method 1: Merton-Perold (MP) Method

The Merton-Perold method directly considers the impact of diversification. It makes use of the fact that the capital required for maintaining several lines of business as one is less than the sum of the capital required for each line of business (assuming of course that the lines are not perfectly correlated). The Merton-Perold method is easy-peezy: the capital allocated to a given line of business is simply given by the amount of capital required for the entire firm, less the capital for the entire firm if that line of business were excluded.

Here is a quick example:

	Required Capital to maintain EPD of 5%	Allocated Capital
Line 1	$1,000	**$500** = 2300 – 1800
Line 2	$1,000	**$900** = 2300 – 1400
Line 3	$1,000	**$600** = 2300 – 1700
Lines 1 & 2	$1,700	
Lines 1 & 3	$1,400	
Lines 2 & 3	$1,800	
Lines 1, 2, & 3	$2,300	

In each, the allocated capital is calculated as the marginal capital required when the excluded business is added to the two-business firm. Note here the total required capital for the 3-line firm is $2,300, while MP assigns only $2,000. That means that $300 is left unassigned to any particular line of business, which is a double-edged sword. It may overstate the estimates of RAROC and EVA, which may cause the firm to take on projects that erode value. However, MP argue that a full allocation of capital may lead the firm to reject value-adding projects.

The underallocation results because the lines of business are not perfectly correlated.

An alternate method of assigning capital using a *fully allocated* means of marginal allocation is the Myers-Read method.

Marginal Capital Method 2: Myers-Read (MR) Method

The Myers-Read method is more of a formulaic beast. In this method, the amount of capital (surplus) assigned to a given line of business, as a ratio of its liability is:

$$s_i = s - \left(\frac{\partial p}{\partial s}\right)^{-1} \left(\frac{\partial p}{\partial \sigma}\right) \frac{[(\sigma_{iL} - \sigma_L^2) - (\sigma_{iV} - \sigma_{LV})]}{\sigma}$$

$s_i =$ surplus − to − liability ratio allocated to line of business i

s = firm surplus-to-liability ratio

p = firm's insolvency put per dollar of total liabilities

σ = firm's overall volatility parameter

$\sigma_{iL} =$ covariance parameter between losses of line i and those of entire liability portfolio

σ_L^2 = volatility parameter for total losses

$\sigma_{iV} =$ covariance parameter between losses of line i and those of entire asset portfolio

σ_{LV} = covariance parameter between the firm's assets and losses

Example (From the Myers-Read paper)

Given the following information:

				Correlations				
Item	Liability	x_i	σ	**Line 1**	**Line 2**	**Line 3**	Cov / L	Cov / V
Line 1	100	33.3%	10.00%	1.000	0.500	0.500	0.0092	-0.0030
Line 2	100	33.3%	15.00%	0.500	1.000	0.500	0.0150	-0.0045
Line 3	100	33.3%	20.00%	0.500	0.500	1.000	0.0217	-0.0060
Liab L	300	100.0%	12.36%	0.742	0.809	0.876	0.0153	-0.0045
Assets	450	150.0%	15.00%	-0.200	-0.200	-0.200		0.0225
Surplus	150	50%						

Variable name from Myers-Read paper	Equivalent variable name in Cummins	Value
σ	σ	21.62817%
d	p	0.311220%
Delta	$\partial p/\partial s$	-0.0237
Vega	$\partial p/\partial \sigma$	0.0838

The values that are relevant for purposes of the calculation of allocation of capital to line 1 are shaded.

We determine capital assigned to line 1 as:

$$s_1 = \left(\frac{150}{300}\right) - (-0.0237)^{-1}(0.0838)\frac{\left[(\mathbf{0.0092} - 0.0153) - \left(-\mathbf{0.0030} - (-0.0045)\right)\right]}{0.2162817}$$

$$s_1 = 0.3758 \rightarrow \$37.58$$

We adjust the bold figures only to similarly arrive at allocations for lines 2 and 3:

$$s_2 = \$49.51; \;\; s_3 = \$62.92$$

The sum of the capital allocated is $150.

A key advantage of the MR method over the MP method is that it always allocates capital exactly. Additionally, it is more intuitively sound, since companies typically adjust lines of business for profitability by adding or removing volume in that business, rather than adding or removing an entire business.

Additional Capital not Considered in Allocations

Each of the above methods allocate capital to line of business, but they do not consider the economic cost of the firm's overall capital, which arises because capital markets are not perfectly efficient. The three most significant contributors to economic cost of capital are:

- **Agency and informational costs:** As you might recall from your Exam 7 readings, agency costs arise when management incentives are not aligned with firm incentives. For example, managers whose bonus is composed of stock options may be less willing to take advantage of opportunities that should benefit the firm, instead choosing riskier investments to try to reap higher rewards.

  Informational costs are borne from the fact that insurance is prone to adverse selection and moral hazard.

- **Double taxation of investment income:** Investors who purchase shares in insurance companies are effectively taxed twice – they are taxed on the investment return of insurers, and the insurers have already been taxed on their investment return. It would be more efficient then for insurance investors to invest directly in the market.

- **Regulatory costs:** Various regulatory constraints can force sub-optimal investment portfolios for insurers. For example, RBC guidelines strongly favor government bonds, so insurance companies invest heavily in those, although they may not compose an optimal portfolio.

These economic costs are important to consider when making a determination as to whether lines of business are generating appropriate rates of return.

Conclusions

Cummins concludes with several points:

- ✓ EPD is more informative than value-at-risk, although value-at-risk does provide helpful knowledge, particularly when calculated at different levels.
- ✓ Option-based models are superior to EPD because they consider the impact of diversification. Of the two models presented, Myers-Read is more in line with how firms actually operate, although it may or may not be more consistent with value-maximization objectives.
- ✓ The cost of capital allocated to a line is the cost in excess of the cost of capital from investing directly in the market.
- ✓ Capital allocation should consider both asset and liability risk, as well as their covariability.
- ✓ Capital allocation should consider duration and maturity of liabilities.
- ✓ The decision-making system should drive data needs, not vice versa.
- ✓ Capital allocation allows firms to make better pricing, underwriting, value-maximization decisions.

OBJECTIVE C REVIEW QUESTIONS

*Questions marked with a **(☆)** contain data in the **Excel Supplement for Exercises**.*

*Some questions also contain solutions in Excel. They are marked with a **(★★)**.*

Coval, Jurek, and Stafford (The Economics of Structured Finance)

CJS-1. Which assumptions did Coval, Jurek, and Stafford find to be most relevant to determining the true value of a structured finance object?

CJS-2. Explain how the senior tranche in a collaterized mortgage obligation (CMO) can be considered similar to a catastrophe bond.

CJS-3. Explain the difference between a mortgage-backed security and a collaterized mortgage obligation. Brownie points if you incorporate the word "overcollaterization."

Exercises 4 – 6 take this format:

Suppose a CDO is created out of two identically-sized assets. The CDO has two tranches – a junior tranche and a senior tranche. In case of default, the asset pays nothing. **The probability of default for each asset is ______% and the assets are________________________.**

Determine the probability of default on an investment in each tranche of the CDO and the CDO^2. "Default" is defined as the investment either partially or completely losing value.

CJS-4. 5%; independent

CJS-5. 5%; perfectly correlated

CJS-6. 20%; independent

CJS-7. Which types of tranches are most (negatively) impacted when correlation assumptions are understated? Explain why this is so.

CJS-8. Explain how CDO^2 tend to magnify the impact of incorrect assumptions.

CJS-9. Explain the role of Freddie Mac and Fannie Mae in the years leading up to the market crisis of 2008.

CJS-10. Provide five reasons behind the incorrect pricing of structured finance objects.

CJS-11. Suppose a CDO is structured as follows:

- 10 bonds in underlying portfolio
- Binary default (assets either default fully or not at all)
- Value of each underlying asset is $1.
- 3 tranches (equity, mezzanine, and senior), where the equity tranche covers the first $2 of defaults, the mezzanine tranche covers the next $2 of defaults, and the senior tranche absorbs any losses thereafter.
- Each asset is independent of all others, and defaults with probability 8%.

a. Determine the probability of default in each tranche. A tranche defaults if any of its assets default.

b. What happens to the probability of default in each tranche if each asset is assumed to be perfectly correlated with all the rest?

CJS-12. Describe the underlying influences behind the creation of the subprime mortgage and the role it caused in the housing bubble.

Cummins (CAT Bond and Other Risk Linked Securities)

Cat-1. Describe why CAT bonds may be preferred over traditional reinsurance (from the insurer standpoint).

Cat-2. Describe why CAT bonds may be less preferred than traditional reinsurance (from the insurer standpoint).

Cat-3. Describe the factors contributing to lack of interest in early CAT bond markets.

Cat-4. In terms of indemnifying the insurer in a given layer of insurer losses, explain how well each of the following mechanisms can guarantee to do so, assuming no counterparty credit risk:

- CAT bonds with an index trigger
- CAT bonds with an indemnity trigger
- Cat-E-Puts
- Catastrophe Risk Swaps
- ILW

Cat-5. What regulatory, accounting, and tax (RAT) issues are identified as impediments to the growth of the CAT bond market?

Cat-6. The CAT bond market has not grown steadily because of several factors. How could the structure of US insurance regulation be improved to allow for additional growth of the market?

Butsic (Solvency Measurement for Property-Liability Risk-Based Capital Applications)

Bu-1. Suppose a new risk-based capital (RBC) model is proposed. This new model would include as one of its risk items an evaluation of the company's CFO. The company's CFO would be scored on a scale of 1 – 10, based in part on the CFO's technical capability, how he is perceived by the public, and his ability to manage big data. Based on only this feature of the proposed RBC model, evaluate whether the model is appropriate.

Bu-2. What is one of the key advantages of the EPD measure over probability-of-ruin?

Bu-3. (Based on Table 1 of the paper) Suppose two insurers A and B each have the same beginning balance sheets ($13,000 in assets), but have liabilities with different distributions, as below:

	Loss Amount; probability
Insurer A	$\begin{cases} 6{,}900;\ p = 0.2 \\ 10{,}000;\ p = 0.6 \\ 13{,}100;\ p = 0.2 \end{cases}$
Insurer B	$\begin{cases} 2{,}000;\ p = 0.2 \\ 10{,}000;\ p = 0.6 \\ 18{,}000;\ p = 0.2 \end{cases}$

a. What is the mean loss of each insurer? How much capital does each hold?

b. Why might the probability-of-ruin criterion be inadequate to measure the policyholders' exposure to loss when comparing these two insurers?

c. Provide another measure that is more appropriate to capture the policyholders' exposure to loss (… and is mentioned in the Butsic paper ☺) and calculate it for both insurers.

Bu-4. An insurer holds assets that will have year-end values and probabilities as below. Determine the probability-of-ruin, the expected deficit, and the EPD ratio, when losses are certain at $10,000.

Assets; probability
$\begin{cases} 15{,}000;\ p = 0.2 \\ 12{,}000;\ p = 0.7 \\ 9{,}000;\ p = 0.1 \end{cases}$

Bu-5. (Based on Table 3 of the paper) Suppose two insurers A and B each have the same expected end-of-year balance sheets ($13,000 in assets), but have liabilities with different distributions, as below:

	Loss Amount; probability
Insurer A	$\begin{cases} 6{,}900;\ p = 0.2 \\ 10{,}000;\ p = 0.6 \\ 13{,}100;\ p = 0.2 \end{cases}$
Insurer B	$\begin{cases} 2{,}000;\ p = 0.2 \\ 10{,}000;\ p = 0.6 \\ 18{,}000;\ p = 0.2 \end{cases}$

Regulatory requirements set capital standards so that EPD ratio is 5%. By how much does each insurer need to adjust their assets to meet this standard?

Bu-6. Insurer A has end-of-year assets expected at $14,000 and liabilities distributed normally with expected value of $10,000 and standard deviation of $1,500.

a. Under this scenario, what is the EPD ratio?

b. What is the probability-of-ruin?

c. If the liabilities were distributed lognormally instead of normally, what would be the EPD ratio?

Bu-7. Repeat the prior exercise, using a standard deviation of 4,500 (so losses are less certain).

Bu-8. Suppose that an insurer's loss profile is given by the following distribution:

Probability	Loss ($Millions)
0.4	$0
0.3	$5
0.2	$10
0.1	$50

a. The insurer holds enough capital to satisfy a 5% expected policyholder deficit (EPD) ratio. How much capital does the insurer require to meet its company requirement?

b. Suppose the insurer wrote another line, independent of the first, with an identical loss profile and entered into a 50% quota share for both products (so the expected loss is still the same). What is the capital requirement now?

Bu-9. An insurer's potential loss profile for two risks is as below:

Probability	Risk A	Risk B
0.5	$20,000	$50,000
0.3	$40,000	$60,000
0.2	$100,000	$90,000

All losses are paid at the end of two years.

To fund the losses, the company invests (at time 0) in assets expected to earn 3.75% per annum.

a. If the company writes only Risk A, determine the initial assets required to satisfy an EPD ratio of 2%.

b. If the company writes only Risk B, determine the initial assets required to satisfy an EPD ratio of 2%.

c. Should the company choose to write both risks, determine the initial assets required to satisfy an EPD ratio of 2%, assuming the risks are perfectly correlated.

d. Should the company choose to write both risks, determine the initial assets required to satisfy an EPD ratio of 2%, assuming the risks are independent.

Bu-10. Fiscella Insurance is reviewing capital adequacy for a certain line of business.

- To that line, it has allocated $45,000 in assets.
- End-of-year assets are expected at $50,000.
- The line has reserves with a mean length of time to payment of 9 years.
- The expected losses are normally distributed with mean $40,000 and standard deviation $20,000.

a. To satisfy regulatory requirements, Fiscella is required to hold at least enough capital to satisfy a 1-year 5% EPD ratio. Determine if Fiscella satisfies regulatory requirements.

b. Explain why the normal distribution may not be appropriate for modelling losses and identify an alternative distribution to use. Without changing the parameters above, determine the EPD ratio under that distribution.

Bu-11. Yiruma Insurance Company projects an unbiased loss reserve that is currently valued at $50,000 for a loss that will be paid at the end of two years.

- Initial assets are $60,000; they are held in an account that credits 5% interest at the end of each year.
- The nature of the loss reserve is such that there is a 20% chance that it increases by 40% at the end of each year, and an 80% chance that it decreases by 10% at the end of each year.

a. Determine the expected policyholder deficit ratio at the end of the first year.

b. Determine the expected policyholder deficit ratio at the end of the second year.

c. Determine the change in capital needed at the beginning of year 2 in order to maintain the same EPD ratio as the year before.

Bu-12. To satisfy management's desires of maintaining a 2% EPD ratio, Johnny Actuary reviews a draft of the company's financial statements to determine asset and liability values. Johnny infers the distribution of expected end-of-year value of the firm's assets based on his knowledge of the portfolio.

Since the bulk of the business is in Worker's Compensation indemnity claims, Johnny uses the expected distribution of claim liability for the duration of the book of business, which is about ten years. Assuming Johnny has correctly pulled each value from the stated sources and appropriately modelled distributions, critique his approach and suggest areas for improvement, where possible.

Bu-13. (Example from the paper) Suppose that we have two independent normally-distributed lines of business, each with a $1,000 expected loss and $200 standard deviation.

a. Show that each line of business in isolation requires $438 in capital in order to maintain a 0.001 EPD ratio.

b. Show that the combined business requires $584 in capital in order to maintain the same ratio.

c. Butsic's **square root rule** to approximate risk capital for independent and partially correlated lines provides a capital equal to the square root of the sum of the squares of each line's required capital, plus their covariance. $\boldsymbol{C_{A+B} = \sqrt{C_A^2 + C_B^2 + 2\rho_{AB} C_A C_B}}$. By how much does the square root rule over or understate the required capital for the joint venture?

Bu-14. Given the following information about some possible 1-year European options, determine the present value of the expected policyholder deficit (EPD) in each scenario. Assume that in each situation, the EPD can be modelled appropriately using the same assumptions as in the option scenarios.

Current Stock Price	500	450
Exercise Price	450	500
Current Value of Call	76.43	21.89
Current Value of Put	14.13	58.22

a. Liabilities are currently estimated at 500 but are unknown at the end of the year. End-of-year assets are known to be 450.

b. Assets are currently estimated at 500 but are unknown at the end of the year. End-of-year liabilities are known to be 450.

c. Liabilities are currently estimated at 450 but are unknown at the end of the year. End-of-year assets are known to be 500.

d. Assets are currently estimated at 450 but are unknown at the end of the year. End-of-year liabilities are known to be 500.

Goldfarb (Risk-Adjusted Performance Measurement for P&C Insurers)

Gol-1. Suppose a firm writes two lines of business in equal volume – personal auto and worker's compensation. Using a return on capital measure, it allocates half of the firm's capital to each line, and each line achieves a 10% return. If the firm were to instead use a RAROC measure to allocate the same total amount of capital, how would you expect the return on each line to change?

Gol-2. Economic profit is one measure of income that ties in closely with how an economist would measure income. What are some of the improvements of this method relative to management's standard GAAP measure of profit, and what are some of its shortfalls?

Gol-3. Firms that are looking to set economic capital, as defined in the Goldfarb paper, may look to set enough capital to meet either a solvency or a capital adequacy objective. Which of the two objectives would likely result in a higher indication of the capital required? Why?

Gol-4. When using a ROC or RAROC measure, firms can opt to set capital based on risk-adjusted or non-risk-adjusted measures. Provide examples and brief descriptions of some risk-adjusted and non-risk-adjusted measures that the firm can consider.

Gol-5. Given that a firm has committed to using probability of ruin as a risk measure, what are three ways it can select the appropriate level to use?

Gol-6. Suppose management has decided they would like to select a default probability based on desire to maintain an AAA-rating. What does the company need to consider in order to determine which probability level to use?

GoI-7. Of the four methods of capital allocation presented in the paper, which would be appropriate in each of the following scenarios?

a. Management is principally concerned with perfect allocations of capital (all capital assigned).

b. The firm is designed in an "all-or-nothing" type scenario where the firm withdraws from the market for any unprofitable business, rather than decreasing volume.

c. The firm wishes to assign the frictional cost of capital.

GoI-8. (Example from Table 29 of the paper) Suppose that a line of business requires 4,225,340 in allocated risk capital. That capital is assumed to be released in the same pattern in which losses are paid, which are on average paid out over four years, with each year paying 50%, 30%, 15%, and 5%, respectively. If the cost of risk capital is 15% and the expected investment income on risk margin is 5%:

a. What is the present value of the cost of risk capital?

b. What is the adjusted target RAROC, after taking into consideration the expected economic profit?

Gol-9. A firm has 5,000,000 in capital to allocate between two lines of business. It runs 1,000 scenarios, with the worst cases by line of business as shown below:

Scenario	Line A	Line B
1000	5,450,000	6,978,000
999	5,423,000	6,838,000
998	5,396,000	6,633,000
997	5,369,000	6,368,000
996	5,342,000	6,113,000
995	5,315,000	5,868,000
994	5,288,000	5,633,000
993	5,262,000	5,408,000
992	5,236,000	5,192,000
991	5,210,000	4,984,000
990	5,184,000	4,785,000
989	5,158,000	4,594,000
988	5,132,000	4,410,000
987	5,106,000	4,234,000
986	5,080,000	4,065,000
985	5,055,000	3,902,000

Determine how much capital should be allocated to each line if the firm allocates proportionately based on:

a. V@R(99.5%)

b. TV@R(99.5%)

c. The expected profit for Line A is 350,000 and the expected profit for Line B is 400,000. Both lines will have the same premium. Management requires a 15% return on capital. Explain what management should consider when deciding which line(s) to write.

Gol-10. A firm is looking to determine the amount of capital needed to support its two lines of business.

a. Describe why the firm may want to consider a risk-adjusted measure of capital rather than a non-risk-adjusted measure.

b. Given that the firm has decided to use a risk-based measure of capital, list some options it could consider.

c. Suppose the firm has opted to use rating agency required capital to determine appropriate capital. To achieve the target rating, a particular agency requires:

- Not more than a 0.3% probability of default
- An EPD ratio of not more than 2%

The firm simulates 1,000 values from the expected aggregate claim distribution, with values shown below. Expected claims are 9,000 and the premium is 11,500.

Scenario	Liability
1000	15,987
999	15,971
998	15,939
997	15,891
996	15,828
995	15,749
994	15,654
993	15,545
992	15,420
991	15,282
990	15,129

Scenario	Liability
989	14,962
988	14,783
987	14,591
986	14,386
985	14,171
984	13,944
983	13,707
982	13,460
981	13,204
980	12,940

Determine how much capital the firm requires.

Gol-11. Determine the actual economic profit for the below line of business, given the following. Assume losses are paid at the end of year 1.

- Premium = 10,000
- Expense Ratio = 10%
- Expected investment return = 8%
- Actual investment return = 12%
- Expected loss ratio = 65%
- Actual loss ratio = 82%

Gol-12. Determine the additional risk margin required for a firm targeting a 15% RAROC, subject to the following assumptions:

- Currently allocated risk capital: 2,500
- Current economic profit: 328
- Expected investment return: 8%
- Expected claims: 750 (paid at time 1)

Gol-13. A firm is working to determine profitability for a line of business it intends to write. The actuary bases his analysis on the assumption of a 12% cost of risk capital. Management has asked that he justify the selection of the 12% assumption. What are some considerations when selecting a target cost of capital?

Gol-14. In the paper, Goldfarb notes the use of several simplifying assumptions he used to present the RAROC method. What are some real-world considerations he disregarded, and how do they impact the measure or understanding of RAROC?

Gol-15. (From the Appendix) Given the following assumptions about a firm considering three sources of risk (Reserves, UW from Line A, UW from Line B):

- Risk Information:

Risk Source	Expected Liability	Cov w/ total Liability
Reserves	18,091,233	0.0141
Line A UW	5,860,732	0.0198
Line B UW	5,860,732	0.0279
Total	29,812,697	n/a

- Firm's capital: 8,949,750
- Liability volatility: 0.1340
- Asset-to-Liability Ratio Volatility: 0.1398
- Delta ($\partial p / \partial s$) = -0.0257
- Vega ($\partial p / \partial \sigma$) = 0.0778
- Assets and liabilities are independent.

Determine the amount of capital allocated to each source of risk, pursuant to the Myers-Read allocation method.

Gol-16. A firm has opted to use probability of ruin to set required capital at a firm-wide level. Describe in detail what considerations apply to this measure, with a specific discussion of the selection of target probability, which risks to include, and the aggregation of the risk portfolio.

Bodoff (Capital Allocation by Percentile Layer)

Bo-1. Explain how capital allocation can be considered similar to an overhead expense. Why is the method chosen to allocate capital important?

Bo-2. What external factors influence how a firm should determine how much capital it needs and what level of return on capital is appropriate?

Bo-3. What are the main drawbacks of the V@R and TV@R methodologies of assigning capital?

Bo-4. What does Bodoff state as the advantages to his capital allocation method?

Bo-5. In the capital allocation method, why do losses in upper layers tend to receive a greater amount of capital allocated than they do in smaller layers?

Bo-6. Suppose a scenario in which there are only two possible perils, as described below:

Peril	Probability	Loss Size
A	25%	40
B	4%	50

Neither event can occur more than once in a year. The firm wishes to allocate capital based on the capital required at the V@R(99) level.

a. How much capital is allocated to each peril using the methodologies that Bodoff coins:
   i. CoVaR
   ii. Alternative CoVaR
   iii. CoTVaR

b. What is the main drawback of each of these methodologies?

Bo-7. **(★★)** For the scenario in the prior example, what would be allocated to each peril using Bodoff's method?

Bo-8. **(★★)** For the scenario in #6, suppose instead that we want to allocate capital consistent with the V@R(96) level. How much capital is allocated to each peril via the CoVar, Alternative CoVar, CoTVaR, and layer methods?

Bo-9. What does a loss's allocated capital depend on?

Bo-10. (Based on Thought Experiment #3) Suppose we have a world with two possible independent perils – Wind that generates a loss of 5M with probability 20%, and Earthquake that generates a loss 100M with probability 5%.

a. What is the mean loss from wind? What is the mean portfolio loss?

b. Assume that the premium is equal exactly to the expected portfolio loss. If a Wind Only event occurs, how much of the premium will it consume?

c. Compare this to the mean loss from wind you found in (a) to explain why it is important to allocate capital to Wind even though its expected value is less than the mean portfolio expected loss.

Bo-11. Suppose that losses are exponentially distributed, with mean 5,000. Suppose that within the distribution, we have loss potentials separated by event, as follows:

- Average Losses are categorized by size 0 – 5,000.
- Bad losses fall between 5,000 and 10,000.
- Catastrophic losses exceed 10,000.

a. Determine the probability of each loss type.

b. Supposing capital is allocated at the V@R(99%) level, determine the amount of capital to be allocated.

c. Determine the amount of capital allocated to each loss type,

   i. If capital is allocated by the "CoVaR" method.

   ii. If capital is allocated by the capital allocation by layer method.

This will be a bit of calculus and algebra, but not as bad as in the uniform distribution.

Bo-12. (Using Bodoff's Thought Experiment) Given a universe with only two possible perils, independent from one another, and summarized as below:

Peril	Loss Size	Probability
Wind	99 MM	20%
Earthquake	100 MM	5%

a. A firm uses a capital requirement at V@R(99%), and uses capital allocation by layer to assign capital to each event. Determine the amount of capital assigned to each peril.

b. Same as (a), but the firm sets capital requirement at TV@R(99%).

c. Determine the net premium required for each of earthquake and wind under the TV@R(99%) capital requirement, assuming an 8% required rate of return.

Bo-13. Losses are exponentially distributed with mean 10,000. A particular loss event, *A*, is categorized by all losses which fall between 5,000 and 20,000.

a. Determine the amount of capital assigned to A if allocating by layer, and required capital is set at the V@R(95.0213) level.

b. Same as (a), but required capital is based on the TV@R(95.0213) level.

c. Same as (b), but we are concerned now with event B, categorized by those losses greater than 20,000.

Cummins (Allocation of Capital in the Insurance Industry)

Cap-1. According to Cummins, why is capital allocation important?

Cap-2. A policy is written at the beginning of the year, and allocated $10 in risk capital. If at the end of the year when the policy expires, it nets $1.8 in income:

a. What is its risk-adjusted rate of return?
b. If the cost of capital is 15%, should the firm have written the policy?
c. What is the economic value added (EVA)?
d. What is the economic value added on capital?
e. Repeat (b) – (d) assuming that the cost of capital is 20%.

Cap-3. According to Cummins, what are the drawbacks and advantages of using RBC to allocate capital?

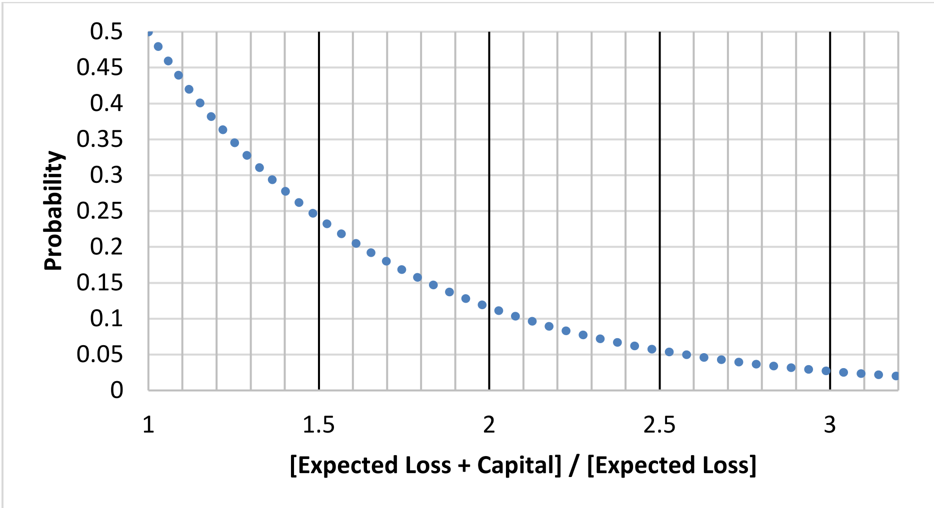
Cap-4. One of Cummins' criticisms of the RBC method is that it does not include all risks. In particular, it excludes derivative risk. What sources of risk *are* included in the RBC model?

Cap-5. Given the following, determine the required rate of underwriting return for the line of business of interest:

- Risk-free rate of return = 5%
- Market rate of return = 13%
- Variance of Market rate of return = 24%
- Covariance between firms' return on equity and market rate of return = 30%
- The beta for the line of interest is 1.2 times the firm's beta.
- Firm equity = 200
- Line's liability = 40

Cap-6. One of the key benefits of the CAPM is that it provides a useful way to view the contributions of a line of business to the firm's ROE, what are some drawbacks of the model?

Cap-7. A given line of business has a V@R exceedance probability curve represented below.



a. A company desires capital enough to shield itself against a 1-in-20 event. If the line of business has an expected loss of $53 million, how much capital should the firm hold, based on the curve?

b. Explain what drawbacks the firm should consider when setting capital using the V@R methodology.

Cap-8. A firm has two lines of business.

- Line A has expected losses of 5,000 and expected end-of-year assets of 6,000.
- Risk-free rate: 5%
- Hurdle Rate: 6.5%
- Here is some information about option pricing:

Strike Price	6,000	5,000	6,000	5,000
Stock Price	5,000	6,000	5,000	6,000
Interest Rate (%)	6.5	6.5	5	5
Value of Call	$98.48	$1,331.09	$83.00	$1,264.80
Value of Put	$720.89	$16.43	$790.38	$20.95

a. Using option pricing methodology, determine the value of Line A's policyholders' claim on the firm, using a one year projection period.

b. Line B has the same parameters as Line A except its volatility is twice as great. Explain (without calculations) the impact on the value of the policyholders' claim for Line B.

c. Explain how the option method of capital allocation can be considered an improvement over the Value-at-Risk method, and also explain what drawbacks it has.

Cap-9. Cummins describes the expected policyholder deficit model as an "insolvency put option." Explain how to view the value of a claim in terms of a put option.

Cap-10. (From Tables 1 – 3 of the paper) A firm is determining how to allocate capital to each line, and determines the below required capital to maintain a 5% EPD ratio for each line alone, as well as for each pair of lines and the entire firm. Each line has an expected liability of $1,000.

Line(s)	Stand-alone capital	Joint Capital
1	361	
2	672	
3	1,107	
1 & 2		745
1 & 3		1,175
2 & 3		1,276
1 & 2 & 3		1,427

a. Explain why the joint capital between each pair of lines is less than the sum of the stand-alone capital required for the lines. In what circumstance would the joint capital exactly equal the sum of the stand-alone capital?

b. Using the Merton-Perold methodology, how much capital does each line of business require? What is the total capital allocated?

Cap-11. **(☆)** Given the following, determine the amount of capital assigned to each line of business, under the Myers-Read method.

Line, i	Liabilities	σ_{iL}	σ_{iV}
A	100	0.0092	-0.0030
B	100	0.015	-0.0045
C	100	0.0217	-0.0060

- Firm surplus: 150
- $\partial p/\partial s = -0.004$
- $\partial p/\partial \sigma = 0.017$
- $\sigma_L^2 = 0.0153$
- $\sigma_{LV} = -0.0045$
- $\sigma = 0.102$

Cap-12. **(☆)** Given the following, determine the amount of capital assigned to each line of business, under the Myers-Read method.

	Value	σ	σ_{iL}	σ_{iV}
Line 1	600	15%	1.87%	0.45%
Line 2	700	15%	1.87%	0.45%
Line 3	800	30%	4.22%	0.90%
Liabilities	2100	16.56%		0.60%
Assets	3000			

$p = 0.16\%;$ $\sigma = 19.49\%$ $Delta\left(\frac{\partial p}{ds}\right) = -0.0147$ $Vega\left(\frac{\partial p}{d\sigma}\right) = 0.0559$

Cap-13. A firm sets capital based on the level representing V@R(99.5). It runs 1,000 scenarios of potential liabilities for line 1 by itself, line 2 by itself, and the combined lines, with the results of the worst scenarios in each run as below:

	Liability		
Scenario	**Line 1**	**Line 2**	**Lines 1 & 2**
1000	900	670	1,500
999	850	650	1,490
998	730	645	1,380
997	700	640	1,300
996	675	620	1,260
995	650	605	1,250
994	650	580	1,240
993	630	530	1,210
992	610	500	1,175
991	605	495	1,100
990	600	420	1,080

Using the Merton-Perold Method, how much of the capital would the firm allocate to each line?

Cap-14. (From Table 3 of the paper) A firm is allocating 1,427 of capital using each of the marginal methods. The results are as follow:

Line(s)	MP Method	MR Method
1	682	811
2	252	392
3	150	224
Total Allocated	**1,084**	**1,427**

Which method is "correct"?

Cap-15. The paper outlines several methods that can be used to allocate capital to individual lines of business but notes that the allocations do not consider the economic cost of capital. What are three sources of economic cost of capital?

OBJECTIVE C REVIEW SOLUTIONS

Coval, Jurek, and Stafford (The Economics of Structured Finance)

CJS-1 Sol. Default and correlation

CJS-2 Sol. The senior tranche of a CMO is very insulated against an individual's default probability. Since a large portion of the individual default risks is absorbed by the lower level tranches, the senior CMO only sees significant losses when a large portion of the underlying risks simultaneously defaults. This is synonymous to a catastrophe bond, which only fails to pay in the event of a major catastrophe. In the senior tranche of a CMO, the catastrophe is essentially the collapse of the housing market.

CJS-3 Sol. An MBS is a type of security that is backed by a mortgage or a collection thereof. This is also known as a mortgage pass through. An MBS allows a bank to quickly liquidate a mortgage by selling it to investors. A CMO is a type of MBS that has different levels of holders. In the case where the underlying mortgages of a CMO are subprime, overcollaterization will occur – the junior tranches will act as a shield to the senior tranches, so the senior tranches are protected not only by collateral from the underlying assets (houses), but from the investors in the junior tranches.

CJS-4 Sol.

Item	When defaults	P(Default)
CDO Junior tranche	At least one asset defaults	$1-(.95)^2 = 9.75\%$
CDO Senior tranche	Both assets default	$(0.05)^2 = .25\%$
CDO^2 Junior tranche	At least one of the four underlying assets defaults	$1-(.95)^4 = 18.5\%$
CDO^2 Senior tranche	Both of the CDOs default	$(1-(.95)^2)^2 = 0.95\%$

CJS-5 Sol.

Item	When defaults	P(Default)
CDO Junior tranche	At least one asset defaults	5%
CDO Senior tranche	Both assets default	
CDO^2 Junior tranche	At least one of the four underlying assets defaults	
CDO^2 Senior tranche	Both of the CDOs default	

When the assets are perfectly correlated, either all or none default.

CJS-6 Sol.

Item	When defaults	P(Default)
CDO Junior tranche	At least one asset defaults	$1-(.8)^2=36\%$
CDO Senior tranche	Both assets default	$(0.2)^2=4\%$
CDO^2 Junior tranche	At least one of the four underlying assets defaults	$1-(.8)^4=59.0\%$
CDO^2 Senior tranche	Both of the CDOs default	$(1-(.8)^2)^2=12.96\%$

CJS-7 Sol. The higher tranches are most negatively impacted. When defaults are independent, the senior tranche only sees a loss when a large portion of underlying assets default. In a large portfolio of completely independent assets, the probability of a large portion of the assets defaulting is very low. In perfect correlation, the chance of a default of all the assets is the same as the chance that any one asset will default.

CJS-8 Sol. CDO^2 take CDOs as their underlying assets. Assumptions for CDOs are then relied upon multiplicatively when synthetic CDOs are constructed. This tends to magnify the impact of those assumptions.

CJS-9 Sol. Since Freddie Mac and Fannie Mae were created to support the mortgage market, the demand for MBS implicitly encouraged banks to accept lower standards in order to fuel the supply of investors. This caused the quality of all mortgage-backed securities to decline. When subprime borrowers started defaulting, the housing market began to collapse, and caused even qualified mortgage holders to default as the underlying assets lost value.

The large default rates meant that the CMOs were overvalued and that large institutions holding a significant portion of their portfolio in these assets quickly lost value.

CJS-10 Sol.

- Failure to consider a decline in housing values.
- Lack of appreciation for rating sensitivity to underlying assumptions.
- Perverse incentives for rating agencies (receive payments from those they rate).
- Perverse incentives for banks (compelled to provide more mortgages due to strength of MBS market).
- Mispricing of tranches involved in CDOs.

CJS-11 Sol.

a. Since it covers first-dollar losses, the equity tranche defaults if at least one asset defaults. The probability of this is given by: $1-(0.92)^{10}=\mathbf{56.6\%}$

The mezzanine tranche defaults if 3 or more of the underlying assets default. This is given by:

$$1-\left[(0.92)^{10}+\binom{10}{1}(0.08)^1(0.92)^9+\binom{10}{2}(0.08)^2(0.92)^8\right]=\mathbf{4.0\%}$$

The senior tranche defaults if at least 5 of the underlying assets default. This is given by:

$$4.0\%-\binom{10}{3}(0.08)^3(0.92)^7-\binom{10}{4}(0.08)^4(0.92)^6=\mathbf{0.06\%}$$

b. If the assets were perfectly correlated, the "good tranches" (e.g., the ones with default probability less than the default probability of each underlying asset) would increase to 8%, while the equity tranche would become much more attractive with a default probability of only 8%.

CJS-12 Sol. Fannie Mae and Freddie Mac were created to encourage home ownership. Due to their governmental entity status, they were able to provide an essentially unlimited amount of cash to banks, removing any capital constraints that would have otherwise required the banks to maintain prudent lending standards.

The ability to "pass through" a mortgage loan allowed banks to continue to have the capital to provide additional mortgages. The popularity of the mortgage-backed security mortgage impelled lending institutions to continue to have a flow of mortgages, which meant lowering credit standards.

With increasingly more mortgages created for credit unworthy borrowers, defaults increased. As defaults increased, the need to sell foreclosed properties to recover collateral did as well. This drove down prices of the assets and caused defaults even among credit worthy borrowers. This tended toward a cycle that ended in a burst bubble.

Cummins (CAT Bond and Other Risk Linked Securities)

Cat-1 Sol.

- CAT bonds are fully collaterized, so the insurer need not worry about the reinsurer defaulting.
- CAT bonds may be used to cover high layers of reinsurance, for which reinsurers may not offer coverage at a reasonable rate.
- Insurers can lock in multi-year protection, unlike in traditional reinsurance.
- CAT bonds may have a lower spread than high-layer reinsurance.

Cat-2 Sol.

- Insurers may not want to rupture long-standing relationships with reinsurers.
- CAT bonds can expose the insurer to basis risk (compensation in case of catastrophe may not align with actual losses), if an indemnity trigger is not used.

Cat-3 Sol. Originally, the market for CAT bonds was very thin. Also, insurers would be concerned with counterparty risk in case of a major catastrophe, and further did not want to risk rupturing long-standing relationships with reinsurers.

Cat-4 Sol. A CAT bond with an index trigger will pay tied to some index not tied to the insurer's losses, so this would not be a full guarantee of coverage in a layer. A CAT bond with an indemnity trigger would payoff directly tied to the losses, so this would provide a full guarantee (given that the insurer purchases the bond for that layer).

Cat-E-Puts would not technically indemnify the insurer for anything – they allow the insurer to raise funds to finance losses (by issuing stock), but they would need to be paid back.

Catastrophe risk swaps would fully guarantee payment in the layer of swap (since we are assuming no credit risk), *if* the payment trigger is met.

Industry loss warranties utilize a dual trigger – a retention trigger that would indemnify the insurer directly, as well as a warranty trigger pegged to the industry. In theory, the presence of the warranty trigger would be problematic if the insurer has substantial losses and the industry does not, but in practice, this is unlikely, so the ILW does for all intents and purposes indemnify the insurer.

Cat-5 Sol.

- Regulatory issues: CAT bonds are typically issued offshore due to favorable costs and expertise levels. Onshore regulators may deny reinsurance accounting treatment for non-indemnity CAT bonds, though bonds may be structured to mimic payoffs for indemnity bonds.
- Tax Issues: Under US tax law, there is no specific information regarding taxation of CAT bonds, so their appropriate treatment is somewhat ambiguous. Currently, CAT bond income is included as bond dividends and not interest income. Some sponsors treat interest similarly to reinsurance premiums.
- Dissemination of information: There is a dearth of publicly available information, which discourages research by potential bond sponsors.

Cat-6 Sol.

- Better reporting: if regulators mandate catastrophe loss reporting in major insurance markets, it would be beneficial for those looking to develop models for CAT bond markets.
- Reinsurance counterparty credit risk should be recognized prior to the loss becoming a recoverable.
- Deregulate prices at the state level so that rates can meet loss expectations.
- Regulators can give credit for multi-year contracts and insurance-linked securities.

Butsic (Solvency Measurement for Property-Liability Risk-Based Capital Applications)

Bu-1 Sol. The desirable characteristics of an RBC model are invariance across classes, objectivity, and ability to differentiate risk. Theoretically, this model should satisfy invariance across classes, as the scoring the CFO does not consider type of insured. The objectivity standard is not met, since measures of "technical capability, public perception, and ability to manage big data" are subjective in nature and would result in different scores depending on the scorer. The model does appear to differentiate risk however; the quality of a CFO would indeed provide indication as to a company's likelihood of remaining solvent.

Overall, the RBC model is *not* appropriate, since it does not meet all three desirable features.

Note: Other answers would be appropriate as well, if well-justified.

Bu-2 Sol. Both measures describe events of insolvency, but probability-of-ruin only determines the likelihood of insolvency, whereas EPD goes further to describe the severity of insolvency.

Bu-3 Sol.

a. Calculate the mean loss as the probability-weighted loss amount. It is **$10,000** for each insurer. Each insurer holds capital of **$3,000** (A = C + L).

b. Both insurers will be insolvent if the third loss occurs (each has 20% probability-of-ruin), but Insurer A will only see a $100 shortfall should it default. Insurer B will have a $5,000 shortfall. Thus, probability-of-ruin does not capture the fact that Insurer B is clearly worse off.

c. I added the parentheticals because there are a number of measures that would be an improvement over probability-of-ruin, but looking for EPD here to be in line with the paper. The expected deficit for Insurer A is 20%(100) = **$20.** The expected deficit for Insurer B is 20%(5,000) = **$1,000.**

   You could instead express these as ratios to expected loss, so EPD Ratio for A and B are **0.2%** and **10%,** respectively.

Bu-4 Sol. The insurer only defaults in the third scenario, when liabilities exceed assets by $1,000.

- The probability-of-ruin is **10%**.
- The expected deficit is 10%($1,000) = **$100**
- The EPD ratio is $100/$10,000 = **1%**

Bu-5 Sol. The probability-weighted loss amount is $10,000 for each insurer, which means that the expected deficit should be 5%($10,000) = $500. The expected deficit currently for insurer A is:

Probability	Loss	Deficit $= \mathbf{MAX(0, Loss - 13,000)}$
0.2	6,900	0
0.6	10,000	0
0.2	13,100	100

Currently, the expected deficit is just 0.2(100) = $20.

We adjust the assets so that (assuming for now that only the third scenario defaults)

0.2(Deficit) = 500 → Deficit = $2,500.

For a deficit of $2,500, when losses are $13,100, assets should be 13,100 – 2,500 = 10,600. Checking back in the second-worst scenario, assets of 10,600 would still not cause a deficit, which is good. So, we confirm that with assets of $10,600, Insurer A's EPD ratio would be 5%. This is a reduction in the current assets by **$2,400.**

Looking now at insurer B:

Probability	Loss	Deficit $= \mathbf{MAX(0, Loss - 13,000)}$
0.2	2,000	0
0.6	10,000	0
0.2	18,000	5,000

Currently, the expected deficit is 0.2(5,000) = $1,000, too big.

Again, we want to see if we can adjust assets so that the third scenario gives a deficit of $2,500.

We want $18,000 – $2,500 = $15,500 in assets. Insurer B should increase its current assets by **$2,500.**

Bu-6 Sol.

a. For the normal distribution, $d_L = k\phi\left(-\frac{c}{k}\right) - c\Phi\left(-\frac{c}{k}\right)$

- k (coefficient of variation) = $\frac{1,500}{10,000} = 15\%$
- c (capital / losses) = $\frac{4,000}{10,000} = 40\%$ (Capital = Assets – Liabilities)
- $d_L = 0.15\phi\left(-\frac{40}{15}\right) - 0.40\Phi\left(-\frac{40}{15}\right) = 0.15 \cdot \frac{1}{\sqrt{2\pi}} e^{-((-40/15)^2/2)} - 0.40(0.0038) =$ $\mathbf{0.019}\%$

b. Probability of ruin is just probability that losses exceed assets. Since this is a normal distribution, we have: $\Phi\left(\frac{10000-14000}{1500}\right) = \Phi\left(-\frac{4000}{1500}\right)$. This is $\Phi\left(-\frac{c}{k}\right)$, which we'd earlier determined as **0.38%.**

c. For lognormal, we use $d_L = \Phi(a) - (1+c)\Phi(a-k)$

- $a = \frac{k}{2} - \frac{\ln(1+c)}{k} = \frac{0.15}{2} - \frac{\ln(1.4)}{0.15} = -2.168$
- $\Phi(-2.168) - (1.4)\Phi(-2.168 - 0.15) = 0.0151 - (1.4)(0.0102) = \mathbf{0.082}\%$

Bu-7 Sol.

a. Same as earlier, except k = 0.45

$$d_L = 0.45\phi\left(-\frac{40}{45}\right) - 0.40\Phi\left(-\frac{40}{45}\right) = 0.45 \cdot \frac{1}{\sqrt{2\pi}} e^{-((40/45)^2/2)} - 0.40(0.187) = \mathbf{4.61}\%$$

b. **18.7%**

c. In this case we use $d_L = \Phi(a) - (1+c)\Phi(a-k)$

- $a = \frac{k}{2} - \frac{\ln(1+c)}{k} = \frac{0.45}{2} - \frac{\ln(1.4)}{0.45} = -0.523$
- $\Phi(-0.523) - (1.4)\Phi(-0.523 - 0.45) = 0.3005 - (1.4)(0.1654) = \mathbf{6.89}\%$

Bu-8 Sol.

a. Expected loss is: $(0.3)(5) + (0.2)(10) + (0.1)(50) = \$8.5$ million

To maintain an EPD ratio of 5%, we need average deficit of 5%(8.5) = $0.425 million.

So if the company defaults only in the last scenario (holds assets of more than $10 million, but less than $50 million), we should have: $0.1(50 - a) = \$0.425 \rightarrow$

$a = \$45.75$ million

The capital required is then $45.75 - 8.5 = \$\mathbf{37.25}$ **million**

b. The combined profile (net of quota share) now looks like:

		Loss A			
		$L_A = 0, p = 0.4$	$L_A = 5, p = 0.3$	$L_A = 10, p = 0.2$	$L_A = 50, p = 0.1$
Loss B	$L_B = 0, p = 0.4$	$L = 0, p = 0.16$	$L = 2.5, p = 0.12$	$L = 5, p = 0.08$	$L = 25, p = 0.04$
	$L_B = 5, p = 0.3$	$L = 2.5, p = 0.12$	$L = 5, p = 0.09$	$L = 7.5, p = 0.06$	$L = 27.5, p = 0.03$
	$L_B = 10, p = 0.2$	$L = 5, p = 0.08$	$L = 7.5, p = 0.06$	$L = 10, p = 0.04$	$L = 30, p = 0.02$
	$L_B = 50, p = 0.1$	$L = 25, p = 0.04$	$L = 27.5, p = 0.03$	$L = 30, p = 0.02$	$L = 50, p = 0.01$

We still want an expected deficit of $0.425 million.

- Suppose $30 \le a < 50$. Then deficit only in last box.
  - $0.01(50 - a) = 0.425 \rightarrow c = 7.5$ (not even close to $30 \le a < 50$)
- Suppose $27.5 \le a < 30$. Then deficit in the bottom corner three boxes.
  - $2(0.02)(30 - a) + 0.01(50 - a) = 0.425 \rightarrow a = 25.5$. Getting closer!
- Suppose $25 \le a < 27.5$. Then deficit in the farthest 3 boxes on the bottom and right.
  - $2(0.03)(27.5 - a) + 2(0.02)(30 - a) + 0.01(50 - a) = 0.425 \rightarrow a = 26.59$

The required capital is now $26.59 - 8.5 = \$\mathbf{18.09}$ **million**

Bu-9 Sol.

a. Expected value for Risk A is $42,000.

   If deficit occurs only in the last scenario, then company is targeting deficit of $0.2(100{,}000 - a) = 2\%(42{,}000) \rightarrow a = \$95{,}800$.

   Since this represents the value of assets at the end of 2 years, required investment is $95{,}800(1.0375)^{-2} = \$\mathbf{89,000}$.

b. Expected value for Risk B is $61,000.

   If deficit occurs only in the last scenario, then company is targeting deficit of $0.2(90{,}000 - a) = 2\%(61{,}000) \rightarrow a = \$83{,}900$.

   Since this represents the value of assets the end of 2 years, required investment is $83{,}900(1.0375)^{-2} = \$\mathbf{77,945}$.

c. If the risks are perfectly correlated, there is no diversification benefit. The required assets are the sum of the assets required for the lines on their own = **$166,944.**

   This is verified by looking for assets satisfying:

$$0.2(190{,}000 - a) = 2\%(103{,}000) \rightarrow a = \$179{,}700$$

   Assets at the beginning of the two-year period would be **$166,944.**

d. If the risks are independent, we would solve for the required assets similar to what was done in the previous problem.

   The expected value is still $103,000, but the distribution will be different.

   Worst events by decreasing severity:

Event	Total Loss	Probability
A3 and B3	$190,000	0.04
A3 and B2	$160,000	0.06
A3 and B1	$150,000	0.1

   Suppose we need capital only in the worst possible event when $a \in [160K, 190000)$.

   Then:

   $0.04(190{,}000 - a) = 2\%(103{,}000) \rightarrow c = \$138{,}500$. Nope.

   Using the worst two events: $c \in [150K, 190000)$

   $0.06(160{,}000 - a) + 0.04(190{,}000 - a) = 2\%(103{,}000) \rightarrow a = \$151{,}400$. Good.

   On inception we would require **$140,653** in assets.

Bu-10 Sol. Currently, Fiscella has capital of $50,000 - $40,000 = $10,000.

a. 9-year liabilities are normal on (40,000; 20,000); 1-year liabilities have a standard deviation of $\sigma_1 = \sqrt{\frac{1}{9} \cdot 20{,}000^2} = 6{,}667$

Then $k = \frac{6{,}667}{40{,}000} = \frac{1}{6}$; $c = \frac{10{,}000}{40{,}000} = 0.25$; $\frac{c}{k} = 1.5$

$d_L = k\phi\left(-\frac{c}{k}\right) - c\Phi\left(-\frac{c}{k}\right) = \frac{1}{6} \cdot \frac{1}{\sqrt{2\pi}} e^{-(1.5^2/2)} - 0.25\Phi(-1.5) = 0.0216 - 0.25(0.0668) =$ $\mathbf{0.489}\%$; **Yes, the company satisfies the 1-year 5% EPD ratio requirement**.

b. The normal distribution assumes that bad outcomes are equally as likely as good outcomes and allows for negative values. In the normal distribution the probability of attaining severely bad outcomes is trivial. This is not true of loss exposures. A better distribution would be the lognormal distribution, which has a heavier tail, and does not allow negative outcomes.

Under the lognormal distribution, we have $d_L = \Phi(\mathrm{a}) - (1 + \mathrm{c})\Phi(a - k)$; $a = \frac{k}{2} - \frac{\ln(1+c)}{k}$

Here, $a = \frac{1}{6} \div 2 - \frac{\ln(1.25)}{1/6} = -1.256$

$$d_L = \Phi(-1.256) - 1.25\Phi\left(-1.256 - \frac{1}{6}\right) = 0.1046 - 1.25(0.0774) = \mathbf{0.785}\%$$

Bu-11 Sol.

a. At the end of the year, assets will be $60,000(1.05) = 63,000. Liabilities will be:

$$L_1 = \begin{cases} 20\% & 70{,}000 \\ 80\% & 45{,}000 \end{cases}$$

Yiruma has a deficit of $7,000 in the first scenario. Expected losses are 70,000(20%) + (45,000)(80%) = 50,000. EPD Ratio = $7000(20\%) \div 50000 = \mathbf{2.8}\%$**.**

b. Continuing the Markov Chain, at the end of year 2, we have four possibilities for end of year 2 liabilities:

Year 0	Year 1		Year 2	
50,000	20%	70,000	4%	98,000
50,000			16%	63,000
50,000	80%	45,000	16%	63,000
50,000			64%	40,500
Expected	**50,000**		**50,000**	

End of year 2 assets will be $60{,}000(1.05)^2 = 66{,}150$.

Yiruma has a deficit of 98,000 – 66,150 = 31,850 in the first scenario. EPD Ratio = $\frac{31{,}850 \cdot 4\%}{50{,}000} =$ $\mathbf{2.55}\%$**.**

c. We want to satisfy $2.8\% = \frac{4\% \cdot \text{Deficit}}{50{,}000} \rightarrow \text{Deficit} = 35{,}000 \rightarrow \text{Assets} = 98{,}000 - 35{,}000 = 63{,}000.$ Beginning of year assets are then $\frac{63{,}000}{1.05} = 60{,}000$ **(capital reduction of $3,000).**

Bu-12 Sol. Johnny should restate the financial sheet items to remove the inherent accounting basis. For purposes of calculating EPD, assets and liabilities should be calculated at market valuation, instead of at the values used for accounting purposes, which may be overly conservative as in STAT valuation, or inconsistent between companies, which may happen with GAAP valuations. Once Johnny has the financial sheet items at market value rates, he should disregard intangible assets such as goodwill, since this also would create an inconsistent measure of RBC.

Another issue Johnny should address is the lack of a common time horizon from which he generated the distributions of assets and liabilities. He used a one-year time horizon for the assets, but a 10-year time horizon for the liabilities. Johnny would do better to use the shorter one-year time horizon, and then periodically update the capital requirements as needed.

Bu-13 Sol.

a. For normally-distributed lines, EPD is determined as $d_L = \frac{D_L}{L} = k\phi\left(-\frac{c}{k}\right) - c\Phi\left(-\frac{c}{k}\right)$

In this case, k = $\frac{200}{1000} = 0.2$ and $c = \frac{438}{1000} = 0.438$

$$d_L = 0.2\phi\left(-\frac{0.438}{0.2}\right) - 0.438\Phi\left(-\frac{0.438}{0.2}\right)$$

$$d_L = 0.2 \cdot \frac{1}{\sqrt{2\pi}} e^{-(2.19)^2/2} - 0.438\Phi(-2.19)$$

$$d_L = 0.00725 - 0.438(.0143)$$

$$d_L = \mathbf{0.001}$$

b. For the combined business, $\mu = 2{,}000$, and

$$\sigma = \sqrt{200^2 + 200^2 + 2(0)(200)(200)} = 282.84$$

$$d_L = \frac{D_L}{L} = k\phi\left(-\frac{c}{k}\right) - c\Phi\left(-\frac{c}{k}\right) = \left(\frac{282.84}{2000}\right)\phi\left(-\frac{584}{282.84}\right) - \left(\frac{584}{2000}\right)\Phi\left(-\frac{584}{282.84}\right)$$

$$= 0.1414\phi(-2.065) - 0.292\Phi(-2.065)$$

$$= 0.1414 \cdot \frac{1}{\sqrt{2\pi}} e^{-(2.065)^2/2} - 0.292(0.0195)$$

$$= \mathbf{0.001}$$

Note that the capital ratio decreased from 0.438 to 0.292.

c. $C_{A+B} = \sqrt{C_A^2 + C_B^2 + 2\rho_{AB}C_AC_B} = \sqrt{438^2 + 438^2 + 2(0)(438)(438)} = 619.$

This is an overstatement of **6.1%** versus the actual 584 required.

Bu-14 Sol.

a. The EPD is the expected excess of liabilities over assets. This is the same as a call option where the underlying security is the liability, currently worth 500 but unknown in the future, and the exercise price represents the assets of 450. (Call option would allow for purchase of the stock for the exercise price; its end-of-year value is the excess of the stock price over the exercise price.) Thus, this is worth **$76.43**.

b. This is the same as a put option where the underlying security is the asset, currently worth 500 but unknown in the future, and the exercise price represents the liabilities of 450. (Put option allows to sell the stock at the exercise price; its end-of-year value is the excess of the exercise price over the stock price.) Thus, this is worth **$14.13**

c. For the same reasoning as in (a), this is worth **$21.89.**

d. For the same reasoning as in (b), this is worth **$58.22**.

Goldfarb (Risk-Adjusted Performance Measurement for P&C Insurers)

Gol-1 Sol. Since the worker's compensation line is inherently riskier than is personal auto due to several factors (duration, lack of data, more volume in excess, volatility, etc.), the risk-adjusted capital assigned to that line of business would increase (and personal auto would decrease). Therefore, I would expect the RAROC of worker's compensation to be < 10%, while the RAROC on auto would increase.

Gol-2 Sol. Economic profit represents an improvement over GAAP because it appropriately values both assets and liabilities, where GAAP does not. Some of the drawbacks are that it still does not capture franchise value, which is a significant portion of the firm's value, and that, because it is not easily-reconcilable to GAAP, its use can complicate management decisions and justifications of decisions to outside users including regulators, rating agencies, and investors.

Gol-3 Sol. The capital adequacy objective looks to provide sufficient capital to maintain future business (dividends, growth). This is a focus on the long-term horizon, and captures franchise value, which is a significant portion of a firm's value. Therefore, the capital adequacy objective would result in a higher indication of required capital than would the solvency objective, which only looks at maintain capital to support existing obligations.

Gol-4 Sol. Non-risk-adjusted measures include:

- Actual committed capital: Provided by shareholders and generates income
- Market Value of Equity: Will exceed committed capital because of inclusion of franchise value

Risk-adjusted measures include:

- Regulatory required capital: for example, RBC
- Rating agency required capital: that required to achieve a certain rating from a particular agency
- Economic capital: Capital required to ensure a specific probability that the firm can achieve a specific objective over a specific time horizon. This can be based on a solvency objective (to meet existing obligations) or a capital adequacy objective (maintain sufficient capital to ensure that firm can pay dividends and support growth over the long term).
- Risk capital: the amount of capital that must be contributed by shareholders to absorb liability risk. This may be the same as economic capital, if there is no risk margin in premiums or reserves.

Gol-5 Sol. The company can use a probability level based on bond ratings (e.g., maintaining an Aa-rating), one based on management preferences, or an arbitrarily selected level targeting a low probability of ruin.

Gol-6 Sol. The firm should consider whether it is targeting a default probability based on probability of being placed into run-off or being downgraded. They should also consider whether the estimates of default rates should be based on historical or current estimates. They should also consider which rating firm to use to define default probability, and which time horizon is applicable.

Gol-7 Sol. The four methods presented in the paper were proportional allocation, incremental allocation, Myers-Read, and co-measures.

a. For perfect allocation, any method other than the Merton-Perold incremental allocation would work.[13]

b. If the firm adds or removes entire business, Merton-Perold incremental allocation would be appropriate.

c. The Myers-Read method assigns frictional cost of capital.

[13] In the Cummins presentation on Merton-Perold, he does not allocate the excess capital (MP does not allocate the excess capital). This is consistent with what was done in MP, and mentioned in a footnote in the Goldfarb paper, though the Goldfarb paper does adjust the allocated capital so that the totals do add up, and notes that in practice there is disagreement over which method is preferable.

Gol-8 Sol.

a. For each year, determine the present value of cost of risk capital based on the cost of the beginning of year capital, discounted using the investment income rate.

Year	% Paid	BOY Capital %	BOY Capital	Cost of Capital (@15%)	PV(Cost of Capital)
1	50%	100%	4,225,340	633,801	603,620
2	30%	50%	2,112,670	316,901	287,438
3	15%	20%	845,068	126,760	109,500
4	5%	5%	211,267	31,690	26,071
Total					**1,026,630**

The total present value is **1,026,630.**

For cost of capital, discounting back to the beginning of year is appropriate since earnings on that capital would have been realized at the end of the year.

b. In the multi-period capital commitment, we adjust the target rate by the ratio of Economic Profit to Initial Capital.

Economic Profit (with $v = \frac{1}{1.05}$) $= 4{,}225{,}340v + 2{,}112{,}670v^2 + 845{,}068v^3 + 211{,}267v^4 = 6{,}844{,}199$

$RAROC = 15\% \cdot \frac{6{,}844{,}199}{4{,}225{,}340} = \mathbf{24.3}\%.$

The target economic profit would be $24.3\%(4{,}225{,}340) = 1{,}026{,}630$ (the present value of cost of capital determined above.)

*See the workup in the directory (Other Files > **Excel Supplements**) for a discussion of how discounting is treated for single v. multiperiods in this paper.*

Gol-9 Sol.

a. We want to hold enough capital to fund all but the 5 worst scenarios. Then we allocate proportionally to sum up to the 5,000,000 in capital.

	V@R(99.5)	Restated
Line A	5,315,000	**2,376,375**
Line B	5,868,000	**2,623,625**
Total	11,183,000	**5,000,000**

b. We are looking for the average of the five worst scenarios.

	TV@R(99.5)	Restated
Line A	5,396,000	**2,251,711**
Line B	6,586,000	**2,748,289**
Total	11,982,000	**5,000,000**

c. Using the V@R method,

$$RAROC_A = \frac{350{,}000}{2{,}376{,}375} = 14.7\%;\ RAROC_B = \frac{400{,}000}{2{,}623{,}625} = 15.2\%$$

Using the TV@R method:

$$RAROC_A = \frac{350{,}000}{2{,}251{,}711} = 15.5\%;\ RAROC_B = \frac{400{,}000}{2{,}748{,}289} = 14.6\%$$

Management's decisions will vary based on the method chosen, and outside factors. Under the V@R method, only B meets the target return. Under the TV@R method, A only would meet.

Gol-10 Sol.

a. A risk-adjusted measure would better reflect the idea that some lines of business should require more allocated capital because they are inherently less predictable than are other lines.

b. It can choose regulatory required capital, rating agency required capital, economic capital, or risk capital.

c. For a 0.3% probability of default, we want to have enough funds for all but the worst $(0.3\%)(1{,}000) = 3$ scenarios, so at least 15,891.

We want to see if this is sufficient to meet the EPD ratio standard:

Scenario	Liability	Total Assets	Deficit
1000	15,987	15,891	96
999	15,971	15,891	80
998	15,939	15,891	48

The expected deficit is $\frac{(96+80+48)\div 1{,}000}{9000} = 0.0025\%$, which is sufficient to meet the standard.

Since the premium is 11,500, the firm must raise a total of 15,891 – 11,500 = 4,391 in additional capital (ignoring any differences in timing).

Gol-11 Sol. The actual economic profit is based off of the actual (not expected) results. The economic profit here is determined as follows:

Premium	10,000	
Expenses	(1,000)	= 10%(10,000)
Investment Income	1,080	= 12%(10,000 – 1,000)
Claims	8,200	= 82%(10,000)
Actual Economic Profit	**1,880**	

Gol-12 Sol. Since we have the economic profit and investment return, we know that: $328 = (P - E)1.08 - 750 \rightarrow P - E = 998.15$

We want economic profit to be at least $15\%(2{,}500) = 375$.

Back-solve: $375 = (998.15 + \pi)1.08 - 750 \rightarrow \boldsymbol{\pi = 43.52}$

Or, using the formula: $\text{RAROC} = \frac{(P+\pi-E)(1+i)-L}{\text{Capital}}$

Using no risk margin for the current economic profit assumption:

$$328 = (P - E)(1.08) - 750 \rightarrow P - E = 998.15$$

To achieve the target, we want: $15\% = \frac{(998.15+\pi)(1.08)-750}{2{,}500} \rightarrow \boldsymbol{\pi = 43.52}$

Gol-13 Sol. The cost of risk capital should consider the way in which RAROC is defined.

- If CAPM was used to establish the cost of risk capital and to assess whether RAROC exceeded it, the actuary should consider that the definitions of risk are different in these measures. CAPM measures systematic risk, while RAROC is concerned with risk between the expected value and values in the tail.
- RAROC is artificially leveraged – the denominator does not reflect market value of invested capital, nor does it reflect the actual capital that could be exposed to loss (committed capital). Shareholders measure return based on the total market value (including franchise value, which is not considered in the RAROC measure), so achieving a 12% return on capital is not necessarily sufficient to meet shareholder's needs.

Gol-14 Sol.

- Time Horizon: There is an inconsistency in the time horizon used to measure different risks. For example, market risk is generally measured over a 1-year period, where insurance liabilities are typically measured over the life of the policy, which may be several years. This can be remedied by measuring market risk over the life of the insurance policy, or to measure the change in value of insurance liabilities over the course of the year.
- Alternative Return Measures: RAROC can be extended to various forms including the following:
  - Firms may choose to use a type of calendar year RAROC calculation, to align with standard management measures.
  - One variation would be to include the impact of taxes on return.
  - Another variation would be to consider the cost of stranded (unallocated) capital.
- Investment income can be reflected in a way other than its present value amount.

Gol-15 Sol. Recall from Cummins that the Myers-Read allocation uses the formula:

$$s_i = s - \left(\frac{\partial p}{\partial s}\right)^{-1}\left(\frac{\partial p}{\partial \sigma}\right)[(\sigma_{iL} - \sigma_L^2) - (\sigma_{iV} - \sigma_{LV})] \cdot \frac{1}{\sigma}$$

Calculation for Reserve risk looks like:

- $s = 8{,}949{,}750 \div 29{,}812{,}697 = 0.3002$
- $\partial p/\partial s = -0.0257$
- $\partial p/\partial \sigma = 0.0778$
- $\sigma_{RL} = 0.0141$
- $\sigma_L^2 = 0.1340^2$
- $\sigma_{RV} = \sigma_{LV} = 0$ (since assets and liabilities are independent)
- $\sigma = 0.1398$
- $s_R = 0.3002 - (-0.0257)^{-1}(0.0778)[0.0141 - 0.1340^2] \cdot \frac{1}{0.1398} = 0.2167$

We allocate $0.2167 \cdot (18{,}091{,}233) = \mathbf{3{,}920{,}392}$ to reserve risk.

With similar calculations, we allocate **1,993,407** to Line A underwriting risk, and **3,021,367** to Line B underwriting risk.

Note that while Lines A and B have the same value for expected liability, Line B is allocated more capital because it has a higher covariance with the total liability.

Total capital allocated here is $3{,}920{,}392 + 1{,}933{,}407 + 3{,}021{,}467 = 8{,}935{,}171$*, 0.16% lower than the total surplus of 8,949,750. Difference is due to rounding of inputs. See the directory (Other Files > **Excel Supplements**) for a spreadsheet workup, where values will hit out exactly.*

Gol-16 Sol. The company needs to consider what to use to determine the target probability.

- They could use that corresponding to a given credit rating level, though they would also need to consider which rating to target. Also, they would need to decide between probability of run-off versus ratings downgrade. Additionally, they should consider whether the probability should be based on the more stable historical estimates of default, or current estimates, which would better reflect current market conditions.
- They could, instead of using a credit rating level, base the probability on the firm's attitude toward risk, though this would differ from the shareholder versus policyholder perspective.
- They could instead choose an arbitrary default probability.

The company also needs to consider which risk sources to incorporate into the aggregate portfolio. These risks may include:

- Market risk (based on changes in equity indices, interest rates, foreign exchange rates, etc.)
- Credit risk (from marketable securities, derivatives, and swap positions, contingent premiums and deductibles, and reinsurance recoverables)
- Insurance Underwriting Risk (including loss reserves from prior policy years, underwriting risk on current policy year, and property catastrophe risk).

The company should also consider how to aggregate its risks. Important notes here are:

- Measuring dependency (for example, based on historical data, subjective estimates, or explicit factor models).
- Aggregating risks after dependency is determined. The company may rely on closed form solutions (though this may be impractical), approximation methods, or simulation methods like copulas.

Bodoff (Capital Allocation by Percentile Layer)

Bo-1 Sol. It affects the entire firm and needs to be allocated by line of business, similar to how overhead is treated. Similarly, the method of allocation can affect profitability, target pricing margins, and volume of business.

Bo-2 Sol. The needs and desires of regulators, rating agencies, and investors.

Bo-3 Sol. V@R considers only one particular loss scenario, that at the given percentile. TV@R goes a step forward in correcting this issue by considering all scenarios above the threshold, calculated as a linear average, but does not assign capital to anything below that level. The V@R methodology looks to hold capital "for a given level of loss," but Bodoff suggests a more appropriate level would be to hold capital sufficient "even for a given level of loss."

Bo-4 Sol. It allocates capital exactly and allocates capital to any event affecting the loss distribution, not just tail events. Additionally, it allocates more capital to events that are either more likely or more severe.

Bo-5 Sol. This is because there are relatively fewer losses that pierce the upper layer, so those that do receive a bigger piece of the pie. Also, losses in the upper layers typically have greater widths due to the shape of the distribution functions appropriate for most insurance applications.

Bo-6 Sol.

a. The sample space of events is:

Event(s)	Probability	Loss Size
None	$(1-0.25)(1-0.04) = 0.72$	0
A only	$(0.25)(1-0.04) = 0.24$	40
B only	$(1-0.25)(0.04) = 0.03$	50
A and B	$(0.25)(0.04) = 0.01$	90

The capital required to cover the V@R(99) level is 50 (1% of losses are above that level). Portion allocated by each method is as follows:

	Portion Allocated to	
Method	Peril A	Peril B
CoVar	**0%**	**100%**
Alternative CoVaR	$\left(\frac{3}{3+1}\right)\left(\frac{0}{50}\right)+\left(\frac{1}{3+1}\right)\left(\frac{40}{90}\right)$ **= 11.1%**	$\left(\frac{3}{3+1}\right)\left(\frac{50}{50}\right)+\left(\frac{1}{3+1}\right)\left(\frac{50}{90}\right)$ **= 88.9%**
CoTVaR	$75\% \cdot 50\left(\frac{0}{50}\right)+25\% \cdot 90\left(\frac{40}{90}\right)=10$ $\frac{10}{60}=\mathbf{16.7}\%$	$75\% \cdot 50\left(\frac{50}{50}\right)+25\% \cdot 90\left(\frac{50}{90}\right)=50$ $\frac{50}{60}=\mathbf{83.3}\%$

b. None of the methodologies assigns terribly much capital to Peril A, although its loss size is nearly as catastrophic and it's even more likely to occur than is Peril B. Also, the expected loss of Peril A is 10, and the expected loss of B is only 2, so it would seem A should have relatively more capital assigned.

Bo-7 Sol. We allocate to each layer based on the contribution of each event to the layer.

Layer	Width	Contribution to Layer (Probability Based)			
		Event 1	Event 2	Event 3	Event 4
0 to 40	40	n/a	$24/28$	$3/28$	$1/28$
40 to 50	10	n/a	n/a	$3/4$	$1/4$

The dollars allocated to each event are:

Event 2: $\frac{24}{28} \times 40 = 34.3$

Event 3: $\frac{3}{28} \times 40 + \frac{3}{4} \times 10 = 11.8$

Event 4: $\frac{1}{28} \times 40 + \frac{1}{4} \times 10 = 3.9$

(Note that the allocations sum to 50, as desired.)

We split up Event 4 by loss size, so Peril A will get $3.9 \times \frac{40}{90} = 1.75$ and Peril B will receive the remaining 2.18.

In total, Peril A receives 34.3 + 1.75 = 36.03.

Peril B will receive 11.8 + 2.18 = 13.97.

Bo-8 Sol. V@R(96) corresponds to a loss of 40.

	Portion Allocated to	
Method	Peril A	Peril B
CoVar A loss of 40 means that only Peril A occurs	**100%**	**0%**
Alternative CoVaR The last three scenarios contribute to capital – use denominator of 28 = 24 + 3 + 1 and assign by relative loss size in each event.	$\frac{1}{28}\left(24\cdot\frac{40}{40}+3\cdot\frac{0}{50}+1\cdot\frac{40}{90}\right)$ **= 87.3%**	$\frac{1}{28}\left(24\cdot\frac{0}{40}+3\cdot\frac{50}{50}+1\cdot\frac{50}{90}\right)$ **= 12.7%**
CoTVaR Similar to above, except also weight by dollars	$\frac{1}{28}(24\cdot 40+3\cdot 0+1\cdot 40)$ **= $35.71** $\frac{35.71}{35.71+7.14}=\mathbf{83.3\%}$	$\frac{1}{28}(24\cdot 0+3\cdot 50+1\cdot 50)$ **= $7.14`** $\mathbf{16.7\%}$

Of note here is that now, each method assigns relatively more capital to Peril A, which is desirable (the capital allocation method would not represent an improvement). In fact, in this case (due to the selection of V@R level), the Alt CoVaR method exactly reproduces the capital allocation by layer assignment.

Bo-9 Sol. It depends on (1) the probability the event occurs, (2) the severity of the event, and (3) the event's inability to share the burden with other losses.

Bo-10 Sol.

a. The mean loss from wind is 5M(20%) = 1M. The mean portfolio loss is 5(20%) + 100(5%) = 6 M

b. It will consume 5/6 (83%) of premium, substantially more than the expected 1M.

c. Even though wind's loss is less than the mean loss of 6M, it can still consume a substantial amount of capital. In this case, it would consume some of the capital that was allocated for the Earthquake event.

Bo-11 Sol. For an exponential function, $f(x) = \frac{1}{\theta} e^{-x/\theta}$ and $F(x) = 1 - e^{-x/\theta}$

In this case: $f(x) = \frac{1}{5000} e^{-x/5000}$; $F(x) = 1 - e^{-x/5000}$

a. A: $1 - e^{-1} = \mathbf{0.632}$

B: $e^{-1} - e^{-2} = \mathbf{0.232}$

C: $e^{-2} = \mathbf{0.135}$

b. $0.99 = 1 - e^{-x/5000} \rightarrow x = \$23{,}026$

c.

i. Only Catastrophic events pierce the threshold, so allocate all to C.

ii. We determine how much is allocated to each event as:

$$AC(x) = \int_{y=0}^{y=\min(23{,}026;\, x)} \frac{f(x)}{1 - F(y)} dy$$

$$= \frac{1}{5{,}000} e^{-x/5000} \int_{y=0}^{y=\min(23{,}026;\, x)} \frac{1}{1 - (1 - e^{-y/5000})} dy$$

$$= \frac{1}{5{,}000} e^{-x/5000} \int_0^{y=\min(23{,}026;\, x)} e^{y/5000} dy$$

$$= \frac{1}{5{,}000} e^{-x/5000} \left[5{,}000 e^{x/5000}\right]_0^{\min(23{,}026;\, x)}$$

$$= e^{-x/5000}\left[e^{\min(23{,}026;\, x)/5000} - 1\right]$$

This is akin to equation A.2 in Appendix A – for exponential distribution, if x< V@R,

$$\boldsymbol{AC(x) = 1 - e^{-x/\theta}}$$

Event	Capital Allocated
A	$\int_{x=0}^{x=5000} (1 - e^{-x/5000})\, dx = \left[x + 5000e^{-x/5000}\right]_0^{5000} = 1{,}839$
B	$\left[x + 5000e^{-x/5000}\right]_{5000}^{10000} = 3{,}837$
C < 23,026	$\left[x + 5000e^{-x/5000}\right]_{10000}^{23026} = 12{,}399$
C > 23,026	$\int_{x=23{,}026}^{x=\infty} \left(e^{(23{,}026-x)/5000} - e^{-x/5000}\right) dx$ $\left[-5{,}000e^{(23{,}026-x)/5000} + 5000e^{-x/5000}\right]_{23{,}026}^{\infty} = 4{,}950$

So, to events A, B, and C, we allocate 1,839; 3,837; and 17,349, respectively. Note that this sums to 23,026.

Just for kicks, if you wanted to use the Horizontal then Vertical approach, which is akin to taking a layer of capital, allocating it across all events, and aggregating that procedure across all layers of capital, the work would look like:

(I'm using $AL(y)$ to represent the losses allocated to capital layer y.)

$$AL(y) = \int_{x=y}^{x=\infty} \frac{f(x)}{1-F(y)} dx$$

$$= e^{y/5000} \int_{x=y}^{x=\infty} \frac{1}{5000} e^{-x/5000} dx$$

$$= e^{y/5000} \left[e^{-x/5000}\right]_{\infty}^{y}$$

$$= e^{y/5000} \left[e^{-y/5000} - 0\right]_{\infty}^{y}$$

$$= 1$$

Taking the second integral, we have:

$$\int_{y=0}^{y=23{,}026} AL(y)dy = \int_{y=0}^{y=23{,}026} 1\, dy = 23{,}026$$

This approach will not tell us how much to assign to an event – it will only confirm that we fully assign each layer to our events.

Bo-12 Sol.

a. This is the problem worked out in the paper. Using it here to tie to the next question.

Event	Loss Size (MM)	Probability
Nothing	0	0.76
W only	99	0.19
E only	100	0.04
W + E	199	0.01

V@R(99) is therefore 100, since there is a 1% chance of exceeding that. Allocating to the loss layers up to 100, we determine amount from each layer allocated to a given event:

	Layer	
Event	**0 – 99**	**99 – 100**
Nothing	0	0
W only	$\frac{.19}{.19+.04+.01} = 0.792$	0
E only	0.167	0.8
W + E	0.042	$\frac{.01}{.01+.04} = 0.2$
Total	**1**	**1**

Assigned to Wind: $0.792(99) + (0.042 \cdot 99 + 0.2 \cdot 1)\left(\frac{99}{199}\right) = \$\mathbf{80.5}$ **MM**

Assigned to Earthquake: $0.167(99) + 0.8(1) + (0.042 \cdot 99 + 0.2 \cdot 1)\left(\frac{100}{199}\right) = \$\mathbf{19.5}$ **MM**

b. The average loss, for those losses above V@R(99%) = 100MM, is 199MM, from the W+E event. So, we are allocating 199MM in capital.

The work will look the same as previously, except we have an additional layer of capital to consider, which is really easy here, since only one event is in that layer.

	Layer		
	0 – 99	**99 – 100**	**100 – 199**
Nothing	0	0	0
W only	$\frac{.19}{.19+.04+.01} = 0.792$	0	0
E only	0.167	0.8	0
W + E	0.042	$\frac{.01}{.01+.04} = 0.2$	1
Total	**1**	**1**	**1**

Assigned to Wind: $0.792(99) + (0.042 \cdot 99 + 0.2 \cdot 1 + 1 \cdot 99)\left(\frac{99}{199}\right) = \$\mathbf{129.8}$ **MM**

Assigned to Earthquake: $0.167(99) + 0.8(1) + (0.042 \cdot 99 + 0.2 \cdot 1 + 1 \cdot 99)\left(\frac{100}{199}\right) = \$\mathbf{69.2}$ **MM**

The total capital allocated in (a) and (b) is 100 and 199, respectively.

> **Note:** Alternately, calculate TV@R(99%) as the weighted average of the losses at least as great as 100, or $\frac{1}{0.05}[0.04(100) + 0.01(199)] = 119.8$
>
> In that case, assign to wind:
>
> $$0.792(99) + (0.042 \cdot 99 + 0.2 \cdot 1 + 1 \cdot 19.8)\left(\frac{99}{199}\right) = \$\mathbf{90.38\ MM}$$
>
> We assign to earthquake:
>
> $$0.167(99) + 0.8(1) + (0.042 \cdot 99 + 0.8 \cdot 1 + 1 \cdot 19.8)\left(\frac{100}{199}\right) = \$\mathbf{29.42\ MM}$$

c. Note that expected loss for wind is $(0.2)(99) = 19.8$; for earthquake it's 5.

Recall that the basic formulation for premium is: $P = E[L] + r(\mathrm{AC} - P)$

Premium for wind: $P = 19.8 + 0.08(129.8 - P) \rightarrow P = \mathbf{27.95}$ **(million)**

You could also use: $P = E[L] + \frac{r}{1+r}(AC - E[L]) = 19.8 + \frac{.08}{1.08}(129.8 - 19.8) = 27.95$

Similarly, earthquake premium will be **9.76 (million).**

Total Premium is 27.95 + 9.76 = 37.70.

Overall expected loss is $99(0.19) + 100(0.04) + 199(0.01) = 24.8$.

Overall capital is 199.

Indicated Overall Premium is $24.8 + \frac{.08}{1.08}(199 - 24.8) = 37.70$*, which ties back to above.*

Bo-13 Sol.

a. V@R(95.0213) is given by x: $1 - e^{-\left(\frac{x}{10000}\right)} = 0.950213 \rightarrow x = \$30{,}000$

(Nice round number justifies my odd choice of V@R level.)

Allocated to our event is then:

$$\int_{x=5{,}000}^{x=20{,}000} \left(1 - e^{-x/10{,}000}\right) dx = \left[x + 10{,}000e^{-x/10{,}000}\right]_{5{,}000}^{20{,}000} = 21{,}354 - 11{,}065$$
$$= \mathbf{10{,}289}$$

b. Since the exponential is a memoryless function, the average excess of losses is the mean. Thus, TV@R(95.0213) = $30,000 + $10,000 = $40,000.

Long way: Algebraically, we calculate the additional layer of capital to be assigned as the average value of losses for those losses in excess of 30,000:

$$\frac{1}{1-0.950213}\int_{30,000}^{\infty}(x-30,0000)f(x)\,dx$$

$$\frac{1}{1-0.950213}\cdot\frac{1}{10,000}\int_{30,000}^{\infty}(x-30,000)\cdot e^{-x/10,000}\,dx$$

Ignoring the denominator:

$$\int_{30,000}^{\infty}\left(xe^{-x/10,000}-30,000e^{-x/10,000}\right)\cdot dx$$

Using integration by parts on the first term gives us:

$$\left[\left(-10,000xe^{-x/10,000}-(10,000)^2e^{-x/10,000}\right)+(30,000)(10,000)e^{-x/10,000}\right]_{30,000}^{\infty}$$

$$\left[-10,000xe^{-x/10,000}+200,000,000e^{-x/10,000}\right]_{30,000}^{\infty}$$

$$10,000(30,000)e^{-3}-200,000,000e^{-3}=4,978,707$$

Bringing back in denominator gives us:

$$\frac{4,978,707}{10,000(1-0.950213)}=10,000$$

The additional $10,000 assigned with the TV@R method will not be affected by this event since it contributes nothing to the $30,000 - $40,000 layer.

Allocated capital is still **$10,289.**

c. We will split the allocation based on those losses in range that fall below or above the required capital of $40,000. We use an equation similar to that derived in question 1(c)(ii).

$$AC(x) = e^{-x/10000}\left[e^{\min(40000;\, x)/10000} - 1\right]$$

$$AC(x) = \begin{cases} 1 - e^{x/10,000} & ; x \leq 40,000 \\ e^{(40,000-x)/10,000} - e^{-x/10,000} & ; x > 40,000 \end{cases}$$

In this case:

$$AC(x) = \int_{x=20,000}^{x=40,000} \left(1 - e^{-x/10,000}\right) dx + \int_{x=40,000}^{x=\infty} \left(e^{(40,000-x)/10,000} - e^{-x/10,000}\right) dx$$

$$= \left[x + 10,000e^{-x/10,000}\right]_{20,000}^{40,000} + \left[-10,000\left(e^{(40,000-x)/10,000} - e^{-x/10,000}\right)\right]_{40,000}^{\infty}$$

$$= 18,830 + 9,817 \text{ (Note that 9,817 is just } 10,000 \times F(40,000)\text{)}$$
$$= \mathbf{28,647}$$

As a check, for losses < 5,000, allocated capital is $\left[x + 10,000e^{-x/10,000}\right]_0^{5,000} = 1,065.$ *Then total allocated capital is $1,065 + $10,289 + $28,647 = $40,000.*

Cummins (Allocation of Capital in the Insurance Industry)

Cap-1 Sol. Capital allocation affects pricing and project selection. Capital allocation is important to policyholders because insolvency risk generally cannot be diversified away.

Cap-2 Sol.

a. $\text{RAROC} = \frac{1.8}{10} = \mathbf{18}\%$

b. Yes; since RAROC is greater than cost of capital, the policy added value.

c. $\text{EVA} = 1.8 - 15\%(10) = \$\mathbf{0.3}$ (Since the EVA is positive, the policy added value.)

d. $\text{EVAOC} = \text{EVA} \div \text{C} = 0.3 \div 10 = \$\mathbf{0.03}$

e. No; in this case, RAROC is less than the cost of capital, so the policy erodes value. Also, the EVA and EVAOC are negative: $1.8 - 20\%(10) = -0.2$, and $\text{EVAOC} = -0.2 \div 10 = -\$0.02$

Cap-3 Sol.

Disadvantages

- The model has little theoretical basis.
- RBC does not consider correlations between lines of business, nor does it consider other significant sources of risk, like risk emanating from derivatives, duration, and convexity.
- RBC charges may not be accurate – they are based on worst-case scenarios and they are based on the average firm, so may not be appropriate for a given non-average firm.
- RBC charges are based on book values rather than market values.

Advantages

- Forces firm to consider regulatory constraints.
- RBC does incorporate some important risks.

Cap-4 Sol. The RBC model includes risks related to holdings from subsidiaries, investment risk, reserve risk, premium risk, credit risk, and off-balance sheet risk.

Cap-5 Sol.

- The firm's beta is $\beta_E = \frac{0.30}{0.24} = 1.25$
- The line's beta is $\beta_i = 1.25 \cdot 1.2 = 1.5$
- The line's liability leverage ratio $k_i = \frac{40}{200} = 0.2$
- The required rate of UW return is:

$$r_i = -k_i r_f + \beta_i(r_M - r_f) = -0.2(5\%) + 1.5(13\% - 5\%) = \mathbf{11}\%$$

Cap-6 Sol.

- The CAPM considers only systematic underwriting risk; it makes no provision for risk stemming from tail events.
- It is difficult to estimate the firm's beta, and it is especially difficult to estimate the underwriting beta for a given line of business.
- The CAPM method has been shown to be insufficient to capture even economic risk (e.g., Fama and French 3-Factor Model).

Cap-7 Sol.

a. A 1-in-20 event is that with 5% chance of occurrence. The point at approximately (2.55, 5%) is the point in which we are interested. The company would need to keep 1.55 times as much capital as the expected loss to meet this tolerance, so $1.55(53) = \$\mathbf{82.15}$ **million.**

b. Issues with V@R are that:

- The firm may not have enough capital to meet this level.
- This approach does not consider the impact of diversification across multiple lines.
- Does not consider the extent of default, given that a loss exceeds the chosen probability level.

Cap-8 Sol.

a. The value of the claim is $Le^{-rt} - \mathrm{P}(A, L, r, \tau, \sigma) = 5{,}000e^{-0.05} - 20.95 = \$\mathbf{4{,}735.20}$. We use the value of the put option at the risk-free interest rate, with strike price equal to the expected liability and stock price equal to the expected assets.

> **Note:** Butsic likens the risky asset-risky liability combination to a put on the current capital, with exercise (strike) price 0, while Cummins uses a put on assets, with exercise price equal to the expected liability. These methods are equivalent.
>
> Following Cummins' approach, you can verify the values in the table using an option pricing calculator. I used http://www.option-price.com/index.php (Accessed 7 Sep 2020.)
>
> With **Price = 6000; Strike = 5000; Days = 365; Interest = 5; Dividend = 0; Volatility = 15** (randomly chosen when I wrote the question), the value of the put option is **20.948**.
>
> Under Butsic's approach, the valuation is more difficult in that we cannot use the Black-Scholes option pricing – the exercise price of 0 will create an undefined value. So instead of using the difference (capital = assets – liabilities), use a quotient. That is, for capital to equal zero (strike price), we must have assets = liabilities. Consider instead the quotient $\frac{A}{L}$, with strike price $= 1$.
>
> Value this with Price = $\frac{A}{L} = \frac{6000}{5000} = 1.2$ and Strike = 1. Using the same values as above for days, interest, dividend, and volatility will yield the same put option value of **20.948**.

b. The put option will increase in value (more volatile, so the insurer is more likely to default). Therefore, the value of the claim will decrease.

c. The insolvency put option (EPD) is more informative than V@R, since it considers the extent of the loss in the tail. It is also consistent with pricing theory. However, it is still difficult to use for purposes of allocating capital by line of business, since each line has the right to access the entire capital of the firm. Additionally, like V@R, the EPD methodology does not consider diversification across lines of business.

Cap-9 Sol. A put option provides an option to sell assets at an agreed upon price. The value of a claim is the present value of the underlying liability, minus the put option. The put option is such that, in the event that liabilities exceed assets, the claimant receives the assets of the firm, rather than the total liabilities due.

Cap-10 Sol.

a. The joint capital is less than the sum because of the impact of diversification. Unless the lines are perfectly correlated, the joint capital will be less than the sum of the stand-alone capitals.

b. For line 1: 1,427 – 1,276 = **151**. For line 2: 1,427 – 1,175 = **252**. For line 3, 1,427 – 745 = **682**. This is a total capital allocation of **1,085**.

Note that the MP method leaves 1,427 – 1,085 = 342 in unallocated capital.

Cap-11 Sol. We use $s_i = s - \left(\frac{\partial p}{\partial s}\right)^{-1}\left(\frac{\partial p}{\partial \sigma}\right)\frac{[(\sigma_{iL}-\sigma_L^2)-(\sigma_{iV}-\sigma_{LV})]}{\sigma}$

Note that $s = \frac{150}{300} = 0.5$ (firm's ratio of surplus to liabilities)

$$s_A = 0.5 - (-0.004)^{-1}(0.017)\frac{[(0.0092-0.0153)-(-0.003--0.0045)]}{0.102} = 0.1833$$

$$s_B = 0.5 - (-0.004)^{-1}(0.017)\frac{[(0.015-0.0153)-(-0.0045--0.0045)]}{0.102} = 0.4875$$

$$s_C = 0.5 - (-0.004)^{-1}(0.017)\frac{[(0.0217-0.0153)-(-0.006--0.0045)]}{0.102} = 0.8292$$

We multiply by each line's respective liabilities to determine final allocations of **18.33, 48.75, and 82.92** to each of lines A, B, and C, respectively. Note that the allocations sum to 150.

Cap-12 Sol. Sample calculation for Line 3 (Note that the firm's capital = 3000 – 2100 = 900.)

$$s_3 = \frac{900}{2100} - (-0.0147)^{-1}(0.0559)\frac{[(0.0422-0.1656^2)-(0.0090-0.0060)]}{0.1949} = 0.658$$

For line 3, the firm holds $0.658(800) = \mathbf{526}$ in capital.

For each of lines 1 and 2, the firm holds a ratio of 0.288 of capital-to-liabilities, so the capital for the respective lines is **172 and 202**. Note that the total capital held is 900, as desired.

Cap-13 Sol. The capital allocated at the V@R(99.5) level is:

- Line 1: 650
- Line 2: 605
- Combined: 1250
- Line 1 would be allocated the total, less that required for line 2 alone:
  1250 – 605 = **645**
- Line 2 would be allocated the total, less that required for line 1 alone:
  1250 – 650 = **600**

  We note the unallocated capital of 5 using this method.

Cap-14 Sol. The more appropriate method depends on the firm's objectives. The Myers-Read method is nice in that it fully allocates capital, and it is more in line with how pricing and underwriting decisions are enacted (by making small changes to an existing portfolio). If the firm is intending to take this approach, the Myers-Read method would be favorable.

If instead the firm is planning on adding entire businesses to the firm, the Merton-Perold method would be more appropriate. Neither method is necessarily superior with regard to consistency with value maximization.

Cap-15 Sol.

- Agency and informational costs – managers may not take actions that are in line with the objective of value maximization, if their compensation is not well-aligned with that goal.
- Double taxation of investment income – investing through an insurance company produces lower returns than directly investing in the market.
- Regulation costs – regulatory requirements may lead to suboptimal portfolio selection.

😊 End of Learning Objective C 😊